Q.1 (4 points) Urn 1 has three red balls and two white balls, and urn 2 has one red ball and five white balls. Let $A$ be the event that a ball is drawn from urn 1, and let $B$ be the event that a red ball is drawn. Suppose that we know $P(A) = \frac{1}{4}$. Then answer the following questions.

(a) Find $P(B)$.

Instead of tossing a coin to decide which urn to choose. You are given $P(A) = \frac{1}{4}$. Otherwise, it goes exactly as in Problem 8.

$$P(B|A)P(A) + P(B|A^c)P(A^c) = \left(\frac{3}{5}\right)\left(\frac{1}{4}\right) + \left(\frac{1}{6}\right)\left(\frac{3}{4}\right) = \frac{11}{40}$$

(b) Find the conditional probability $P(A|B)$.

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B|A)P(A)}{P(B)} = \frac{(3/5)(1/4)}{(11/40)} = \frac{6}{11}$$

Q.2 (6 points) Let $B$ and $D$ be independent events with $P(B) = \frac{1}{5}$ and $P(D) = \frac{1}{3}$.

(a) Find $P(B \cap D)$.

Since $B$ and $D$ are independent, we have $P(B \cap D) = P(B)P(D) = \frac{1}{15}$

(b) Find $P(B \cup D)$.

$$P(B \cup D) = P(B) + P(D) - P(B \cap D) = \frac{1}{5} + \frac{1}{3} - \frac{1}{15} = \frac{7}{15}$$

(c) Find the conditional probability $P(B|B \cup D)$.

Problem 7 similarly uses the definition of conditional probability. But you must also notice $B \cap (B \cup D) = B$. Then we can find

$$P(B|B \cup D) = \frac{P(B)}{P(B \cup D)} = \frac{1/5}{7/15} = \frac{3}{7}$$