Instructions: Start each question on a separate sheet of paper. Show all work to maximize any partial credit. Practice under test conditions ... no notes, book, or calculator!

For questions 1, 2, and 3 use the following system of linear equations.

\[
\begin{align*}
x_1 + hx_2 &= 2 \\
4x_1 + 8x_2 &= k
\end{align*}
\]

1. For what values of \(h\) and \(k\) is the system inconsistent? (10 pts)

2. For what values of \(h\) and \(k\) does the system have a unique solution? (15 pts)

3. For what values of \(h\) and \(k\) does the system have many solutions? (10 pts)

For questions 4, 5, and 6, use the augmented matrix given by

\[
\begin{bmatrix}
2 & 3 & -4 & 17 & -5 \\
-3 & -3 & 4 & 10 & 6 \\
4 & 1 & 4 & 0 & 3 \\
\end{bmatrix}
\]

4. Write the system of linear equations represented by the augmented matrix. (5 pts)

5. Write the system of linear equations represented by the augmented matrix as a vector equation. (10 pts)

6. Write the system of linear equations represented by the augmented matrix in the form \(Ax = b\). (10 pts)

7. Given matrix \(A\) below, does the equation \(Ax = b\) have a solution for every \(b\) in \(\mathbb{R}^4\)? Show all work justifying your answer with the appropriate details. (15 pts)

\[
A = \begin{bmatrix}
0 & 0 & 0 & 1 \\
0 & 1 & 0 & 0 \\
0 & 0 & 3 & 3 \\
4 & 0 & 0 & 0 \\
\end{bmatrix}
\]

8. Find the general solution to the system of equations whose augmented matrix is below. Indicate which variables are free (if any). (10 pts)

\[
A = \begin{bmatrix}
1 & -3 & 0 & -1 & 0 & -2 \\
0 & 1 & 0 & 0 & -4 & 1 \\
0 & 0 & 0 & 1 & 9 & 4 \\
0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

9. For each of the statements below, indicate True or False. (3 pts ea)

- Every elementary row operation is reversible.
- Elementary row operations on an augmented matrix never change the solution set of the associated linear system.
- The echelon form of a matrix is unique.
- If the coefficient matrix of a system of equations has a pivot position in every column then the system is consistent.
- Asking whether the linear system corresponding to an augmented matrix \([a_1 \ a_2 \ a_3 \ b]\) has a solution amounts to asking whether \(b\) is in the \(Span\{a_1, \ a_2, \ a_3\}\).