ON THE SEPARATING IDEALS OF SOME VECTOR-VALUED GROUP ALGEBRAS

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Abstract. For a locally compact Abelian group $G$, and a commutative Banach algebra $B$, let $L^1(G, B)$ be the Banach algebra of all Bochner integrable functions. We show that if $G$ is noncompact and $B$ is a semiprime Banach algebras in which every minimal prime ideal is contained in a regular maximal ideal, then $L^1(G, B)$ contains no nontrivial separating ideal. As a consequence we deduce some automatic continuity results for $L^1(G, B)$.

1. INTRODUCTION. For any locally compact Abelian group $G$, and commutative Banach algebra $B$, let $L^1(G, B)$ denote the convolution algebra of all integrable functions on $G$ with values in $B$. As one might expect, there are some interesting similarities between $B$ and $L^1(G, B)$. For instance, $L^1(G, B)$ is semi-simple if and only if $B$ is semi-simple, and the regular maximal ideals of $L^1(G, B)$ are closely related in a natural way with the regular maximal ideals of both $L^1(G, B)$ and $B$. Also, $L^1(G, B)$ is Tauberian if and only if $B$ is Tauberian. Refer to [8,9] for the proofs of the above results. Also it is easy to note that $L^1(G, B)$ is semiprime when $B$ is semiprime. The question whether the zero ideal is the only separating ideal in a semiprime Banach algebra still seems to be open. However, in this paper we prove that when $G$ is a noncompact locally compact Abelian group, and $B$ is a commutative semiprime Banach algebra (not necessarily unital) in which every minimal prime ideal is contained in a regular maximal ideal, then $L^1(G, B)$ contains no non-trivial separating ideal. As a consequence we deduce some automatic continuity results for the algebra $L^1(G, B)$. Our results extend some of the results in [11] for non unital Banach algebras, and also extend some results in [7] for semiprime Banach algebras. For relevant information on $L^1(G, B)$ and for related results in harmonic analysis on Abelian groups,
2. PRELIMINARIES. Let $B$ be a commutative Banach algebra (not necessarily unital), and let $G$ be a locally compact Abelian group with Haar measure $m$. Throughout the following, the dual group of $G$ is denoted by $\Gamma$ and the spectrum of $B$ is denoted by $\Delta(B)$. Let $L^1(G, B)$ denote the Banach algebra of all integrable function from $G$ into $B$,

\[(f * g)(t) := \int G f(t - s)g(s)dm \quad \text{for all } f, g \in L^1(G, B) \text{ and } t \in G,
\]

and let $\|f\|_1 := \int G \|f(t)\|dm(t)$ for all $f \in L^1(G, B)$. Recall that for any $f \in L^1(G, B)$, and $\gamma$ in the dual group $\Gamma$ of $G$, $\hat{f}(\gamma) = \int G \gamma(t)f(t)dm(t)$ is known as the vector-valued Fourier transform of $f$ at $\gamma$. Furthermore for any $\gamma \in \Gamma$, let $M_\gamma := \{f \in L^1(G, B) : \hat{f}(\gamma) = \theta\}$ where $\theta$ is the zero vector of $B$. Clearly, $M_\gamma$ is a closed ideal of $L^1(G, B)$. If $B$ has no non-trivial zero divisors, then $M_\gamma$ is a closed prime ideal of $L^1(G, B)$. Recall that an ideal $I$ of a commutative Banach algebra is said to be prime if the product $xy \in I$ only if either $x \in I$ or $y \in I$. It is an easy consequence of the Hahn-Banach theorem that $\bigcap_{\gamma \in \Gamma} M_\gamma$ is the zero ideal in $L^1(G, B)$. For any $\gamma \in \Gamma$, $\phi \in \Delta(B)$, let

\[M_{\gamma, \phi} := \{f \in L^1(G, B) | \phi(\hat{f}(\gamma)) = 0\}.
\]

The regular maximal ideals of $L^1(G, B)$ are given by $M_{\gamma, \phi}$ for some $\gamma \in \Gamma$, and $\phi \in \Delta(B)$ ([8]).

For each $f \in L^1(G)$, and $x \in B$, we let

\[(f \otimes x)(s) = f(s)x \quad \text{for all } s \in G.
\]

We recall some of the properties of the product $f \otimes x$ in the following proposition.

**Proposition 2.1.** Let $G$ be a locally compact Abelian group, and let $B$ be a commutative Banach algebra. Let $x, y \in B$; $f, g \in L^1(G)$; and $\gamma$ a non-trivial continuous character on $G$. Then,
(i) $f \otimes x \in L^1(G, B)$, and $\|f \otimes x\|_1 = \|f\|_1 \|x\|

(ii) $(f \pm g) \otimes x = f \otimes x \pm g \otimes x$

(iii) $\gamma \otimes x(\gamma) = \hat{f}(\gamma)x$

(iv) $(f \otimes x) * (g \otimes x) = (f * g) \otimes xy$

(v) If $B$ has the multiplicative identity $1$, then $(f * g) \otimes x = (f \otimes x) * (g \otimes 1) = (f \otimes 1) * (g \otimes x)$

(vi) If $f_n \to f$ in $L^1(G)$ and $x_n \to x$ in $B$, then $f_n \otimes x_n \to f \otimes x$ in $L^1(G, B)$.

3. Main Results

Before we get to the main results, we need the following lemmas.

**Lemma 3.1.** Let $G$ be a noncompact locally compact Abelian group, $B$ a commutative Banach algebra, and $f$ a non-zero function in $L^1(G, B)$. For a given $\gamma$ in the dual group $\Gamma$ of $G$ and a positive number $\varepsilon$, there exist $f_1, f_2, \ldots, f_n$ in $L^1(G)$ with compactly supported Fourier transforms and $x_1, x_2, \ldots, x_n$ in $B$ such that $\|f - \sum_{i=1}^{n} f_i \otimes x_i\| < \varepsilon + \|\hat{f}(\gamma)\|$, where $\hat{f}_i(\gamma) = 0$ for $1 \leq i \leq n$.

**Proof.** Since finite linear combinations of the elements of the form $h \otimes x$ where $h \in L^1(G)$, and $x \in B$ are dense in $L^1(G)$, and the functions in $L^1(G)$ with compactly supported Fourier transforms are dense in $L^1(G)$, there exist $h_1, h_2, \ldots, h_n$ in $L^1(G)$ with compactly supported Fourier transforms and $x_1, x_2, \ldots, x_n$ in $B$ such that $\|f - \sum_{i=1}^{n} h_i \otimes x_i\| < \frac{\varepsilon}{2}$. For $1 \leq i \leq n$, let $\text{Supp } \hat{h}_i = \{\alpha \in \Gamma : \hat{h}_i(\alpha) \neq 0\}$. For each $1 \leq i \leq n$, we define

$$g(t) = \frac{\chi_{(\bigcup_{j=1}^{n} \text{Supp } \hat{h}_j) \gamma(t)} \chi_{(\bigcup_{j=1}^{n} \text{Supp } \hat{h}_i)}}{m(\bigcup_{j=1}^{n} \text{Supp } \hat{h}_i)}$$

where $\chi_{(\bigcup_{j=1}^{n} \text{Supp } \hat{h}_j)}$ is the characteristic function of $(\bigcup_{j=1}^{n} \text{Supp } \hat{h}_j)$, and $f_i = h_i - \hat{h}_i(\gamma)g$. Clearly $g$ and the $f_i$'s belong to $L^1(G)$. It is easy to see that $\hat{g}(\gamma) = 1, \hat{f}_i(\gamma) = 0$ for each
$i$, and $\|g\|_1 = 1$. We have

$$
\|f - \sum_{i=1}^{N} f_i \otimes x_i - g \otimes \hat{f}(\gamma)\|_X = \|f - \sum_{i=1}^{N} (h_i \otimes x_i) - (f_i \otimes x_i) - g \otimes \hat{f}(\gamma)\|_X
\leq \|f - \sum_{i=1}^{N} (h_i \otimes x_i)\|_X + \sum_{i=1}^{N} \| (h_i - f_i) \otimes x_i - g \otimes \hat{f}(\gamma)\|_X \quad \ldots (A)
$$

Furthermore,

$$
\|\sum_{i=1}^{N} (h_i - f_i) \otimes x_i - g \otimes \hat{f}(\gamma)\|_X = \|\sum_{i=1}^{N} \hat{h}_i(\gamma) g \otimes s_i - g \otimes \hat{f}(\gamma)\|_X
\leq \|\sum_{i=1}^{N} \hat{h}_i(\gamma) g(t)x_i - g(t)\hat{f}(\gamma)\|_X dm(t)
\leq \|\sum_{i=1}^{N} \hat{h}_i(\gamma) g(t)x_i - \hat{f}(\gamma)\|_X dm(t)
\leq \|\sum_{i=1}^{N} \hat{h}_i(\gamma) g(t)x_i - \hat{f}(\gamma)\|_X \leq \frac{\varepsilon}{2} \quad \ldots (B)
$$

From (A) and (B) it follows that $\|F - \sum_{i=1}^{N} f_i \otimes x_i - g \otimes \hat{f}(\gamma)\|_X < \varepsilon$. Hence $\|f - \sum_{i=1}^{N} f_i \otimes x_i\| < \varepsilon + \|\hat{f}(\gamma)\|$. This completes the proof of the Lemma.

\textbf{Lemma 3.2.} Let $G$ be a noncompact locally compact Abelian group, $B$ a commutative Banach algebra, and $f$ a non-zero function in $L^1(G, B)$. For a given $\gamma$ in the dual group $\Gamma$ of $G$ and a given positive number $\varepsilon > 0$, there exist $g_1, g_2, \ldots, g_n$ in $L^1(G)$, a neighborhood $V$ of $\gamma$, and $x_1, x_2, \ldots, x_n$ in $B$ such that

$$
\|f - \sum_{i=1}^{N} g_i \otimes x_i\| < \varepsilon + \|\hat{f}(\gamma)\|
$$

where $\hat{g}_i = 0$ on $V$ for $1 \leq i \leq n$. 
Proof. By Lemma 3.1, there exist \( f_1, f_2, \ldots, f_n \) in \( L^1(G) \) with compactly supported Fourier transforms, and \( x_1, x_2, \ldots, x_n \) in \( B \) such that

\[
\| f - \sum_{i=1}^{X^n} f_i \otimes x_i \| < \frac{\epsilon}{2} + \| \hat{f}(\gamma) \|
\]

where \( \hat{f}_i(\gamma) = 0 \). Since \( L^1(G) \) satisfies the Ditkin’s condition ([12]), there exist \( g_1, g_2, \ldots, g_n \) in \( L^1(G) \), and a neighborhood \( V \) of \( \gamma \) such that \( \hat{g}_i = 0 \) on \( V \), and

\[
\| f_i - g_i \|_1 < \frac{\epsilon}{2(1 + \| x_i \|)}
\]

for \( 1 \leq i \leq n \). Now

\[
\| f - \sum_{i=1}^{X^n} g_i \otimes x_i \| \leq \| f - \sum_{i=1}^{X^n} f_i \otimes x_i \| + \| (f_i - g_i) \otimes x_i \|
\]

\[
\leq \frac{\epsilon}{2} + \| \hat{f}(\gamma) \| + \sum_{i=1}^{X^n} \| f_i - g_i \|_1 \| x_i \|
\]

\[
< \frac{\epsilon}{2} + \| \hat{f}(\gamma) \| + \frac{\epsilon}{2(1 + \| x_i \|)} \sum_{i=1}^{X^n} \| x_i \|
\]

\[
= \epsilon + \| \hat{f}(\gamma) \|.
\]

Corollary 3.3. Let \( f \in L^1(G, B) \), and \( \gamma \in \Gamma \) such that \( \hat{f}(\gamma) = \theta \). Given \( \epsilon > 0 \), there exist \( g_1, g_2, \ldots, g_n \) in \( L^1(G) \) with a vanishing Fourier transform in a neighborhood \( V \) of \( \gamma \), and \( x_1, x_2, \ldots, x_n \) in \( B \) such that \( \| f - \sum_{i=1}^{X^n} g_i \otimes x_i \| < \epsilon \).

Proof. Obviously follows from the Lemma 3.2.

Now we are ready for the main results of the section.

Theorem 3.4 Let \( G \) be a locally compact Abelian group, \( \gamma \) a continuous character on \( G \), and \( \mathcal{P} \) a prime ideal contained in \( M_\gamma \). Then \( \mathcal{P} \) is dense in \( M_\gamma \).

Proof. Let \( \mathcal{P} \) be a prime ideal of \( L^1(G, B) \) contained in \( M_\gamma \). Let \( f \) be a function with \( \hat{f} \) identically equal to the zero vector in a neighborhood \( V \) of \( \gamma \). We claim that \( f \) belongs
to \( P \). For, if \( g \) belongs to \( L^1(G) \) with \( \hat{g}(\gamma) \neq 0 \), \( \hat{g} = 0 \) on \( \Gamma - V \), and \( x \) a non-zero vector in \( B \), then \((g \otimes x) * f = \Theta \) (the zero vector of \( L^1(G, B) \)). Since \( P \) is a prime ideal of \( L^1(G, B) \), either \( g \otimes x \in P \) or \( f \in P \). But \( g \otimes x(\gamma) = \hat{g}(\gamma)x \neq 0 \). Hence \( f \in P \). Thus all the functions \( f \) in \( L^1(G, B) \) with vanishing Fourier transforms in a neighborhood of \( \gamma \) belong to \( P \). Hence by Lemma 3.2, it follows that \( P \) is dense in \( M_\gamma \). This completes the proof of the theorem.

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**Theorem 3.5.** Let \( G \) be a noncompact locally compact Abelian group, and \( B \) be a commutative Banach algebra. If \( P \) is a closed prime ideal of \( L^1(G, B) \) contained in \( M_{\gamma, \phi} \) for some \( \gamma \in \Gamma \), and \( \phi \in \Delta(B) \), then \( P \) contains \( M_\gamma \). Furthermore \( P \) does not contain \( M_\sigma \) for any \( \sigma \neq \gamma \).

**Proof.** Let \( f \in M_\gamma \). By Corollary 3.3, \( f \) can be approximated by a function \( g \) in \( L^1(G, B) \) with vanishing Fourier transform in a neighborhood \( V \) of \( \gamma \). By an argument similar to the one given in Theorem 3.4, we can show \( g \in P \). Since \( P \) is a closed ideal, it follows that \( f \in P \). Thus \( M_\gamma \) is contained in \( P \). Let \( \sigma \in \Gamma \) such that \( \sigma \neq \gamma \). Suppose \( V_\sigma \) and \( V_\gamma \) are compact neighborhoods of \( \sigma \) and \( \gamma \) respectively such \( V_\sigma \cap V_\gamma = \emptyset \). Then there exist functions \( f_\sigma \) and \( f_\gamma \) from \( G \) into the complex plane with the support of \( \hat{f}_\sigma \) contained in \( V_\sigma \) and the support of \( \hat{f}_\gamma \) contained in \( V_\gamma \) such that \( \hat{f}_\sigma(\sigma) = 1 \) and \( \hat{f}_\gamma(\gamma) = 1 \). Let \( x, y \in B \) such that \( \phi(x) \phi(y) \neq 0 \). Then \( f_\sigma \otimes x, f_\gamma \otimes y \in L^1(G, B) \) such that \((f_\sigma \otimes x) * (f_\sigma \otimes y) = \Theta \). Since \( P \) is a prime ideal contained in \( M_{\gamma, \sigma} \), we get \( f_\sigma \otimes x \in P \). Obviously \( f_\gamma \otimes y \notin P \). However \( f_\gamma \otimes y \in M_\sigma \). Therefore \( M_\sigma \) is not contained in \( P \).

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**4. Applications.**

Recall that a closed ideal \( S \) of a commutative Banach algebra \( A \) is called a separating ideal ([3]) if it satisfies the following condition: For each sequence \( \{a_k\}_{k \geq 1} \) in \( A \) there is a positive integer \( n \) such that \( a_1a_2 \cdots a_nS = a_1a_2 \cdots a_kS \) \((k \geq n)\). For any derivation \( D \) on \( A \), let \( \Im(D) =: \{a \in A | \text{there is a sequence } \{a_n\} \text{ in } A \text{ with } a_n \to 0 \text{ and } Da_n \to a\} \). For any
epimorphism \( h \) form a commutative Banach algebra \( X \) onto \( A \), let \( \exists(h) = \{ a \in A \mid \text{there is a sequence } \{x_n\} \text{ in } X \text{ with } x_n \to 0 \text{ and } h(x_n) \to a \} \). It is easy to show that \( \exists(D), \exists(h) \) are closed ideals of \( A \). By the closed graph theorem \( D \) is continuous if and only if \( \exists(D) \) is zero. Similarly \( h \) is continuous if and only if \( \exists(h) \) is zero. It is well known that \( \exists(D) \) and \( \exists(h) \) are separating ideals of \( A \) ([13]). For further information on separating ideals, their relation to the prime ideals of the Banach algebra, and for related results on automatic continuity theory, see [1,2,3,4,6,10].

Now we are ready to state one of the main results of the section.

**Theorem 4.1.** Let \( G \) be a noncompact locally compact Abelian group \( G \), and \( B \) a commutative semiprime Banach algebra in which every minimal prime ideal is contained in a regular maximal ideal. Then \( L^1(G, B) \) contains no nontrivial separating ideal.

**Lemma 4.2.** Let \( G \) be a noncompact locally compact Abelian group \( G \), and \( B \) a commutative semiprime Banach algebra. For any \( \gamma \in \Gamma, \ M_\gamma = \bigcap_{P \in I_\gamma} P \) where \( I_\gamma \) is the set of all minimal prime ideals of \( L^1(G, B) \) containing \( M_\gamma \).

**Proof.** Let \( f \in \bigcap_{P \in I_\gamma} P \). Since there is a one-to-one correspondence between the prime ideals of the quotient algebra \( L^1(G, B)/M_\gamma \) and the prime ideals of the algebra \( L^1(G, B) \) containing \( M_\gamma \), there exists a positive integer \( n \) such that \( f * f * \cdots * f \in M_\gamma \). This implies \((\hat{f}(\gamma))^n = \theta \). Since \( B \) is semiprime, \( \hat{f}(\gamma) = \theta \). Hence \( f \in M_\gamma \).

**Proof of Theorem 4.1.** If possible assume that \( \exists \) is a nontrivial separating ideal in \( L^1(G, B) \).

**Claim.** \( \exists \) is contained in all but finitely many \( M_\gamma \) for \( \gamma \in \Gamma \).

**Proof of the claim.** Let \( \mathcal{M} \) be the set of all minimal prime ideals of \( L^1(G, B) \) not containing \( \exists \). By [3] \( \mathcal{M} \) is a finite set. Let

\[
\mathcal{M}_\Delta = \{ P \in \mathcal{M} \mid P \subseteq M_{\gamma_0} \text{ for some } (\gamma, \phi) \in \Gamma \times \Delta(B) \}
\]

and \( \mathcal{M}_\Delta^0 = \mathcal{M} - \mathcal{M}_\Delta \). By Theorem 3.5, each member of \( \mathcal{M}_\Delta \) contains a unique \( M_\gamma \) for...
some $\gamma \in \Gamma$. Let $\Gamma_{\Delta} = \{ \gamma \in \Gamma | M_\gamma \subseteq P \text{ for some } P \in M_\Delta \}$. Obviously $\Gamma_{\Delta}$ is a finite set. Since $\exists$ is contained in all but finitely many closed prime ideals of $L^1(G, B)$ ([3]), and since any prime ideal contains a minimal prime ideal, it follows that $\Gamma_{\Delta}$ is not empty. Let $\gamma \in \Gamma - \Gamma_{\Delta}$. By Lemma 4.2, $M_\gamma = \bigcap_{P \in \mathcal{I}_\gamma} P$ where $\mathcal{I}_\gamma$ is the set consisting of all minimal prime ideals of $L^1(G, B)$ containing $M_\gamma$. Write $\mathcal{I}_\gamma = \mathcal{I}_\Delta \cup \mathcal{I}_{\Delta_0} \cup \mathcal{I}_{\Delta_00}$ where

$$\mathcal{I}_\Delta = \{ P \in \mathcal{I}_\gamma | P \subseteq M_{\gamma, \phi} \text{ for some } \phi \in \Delta(B) \},$$

$$\mathcal{I}_{\Delta_0} = \{ P \in \mathcal{I}_\gamma | P \text{ contains } \exists, \text{ and } P \nsubseteq M_{\gamma, \phi} \text{ for each } \phi \in \Delta(B) \}$$

and

$$\mathcal{I}_{\Delta_00} = \{ P \in \mathcal{I}_\gamma | P \text{ does not contain } \exists \text{ and } P \nsubseteq M_{\gamma, \phi} \text{ for each } \phi \in \Delta(B) \}.$$ 

Notice that $\mathcal{I}_{\Delta_0}$ is almost a finite set, and each $P$ in $\mathcal{I}_\Delta$ contains $\exists$. Obviously

$$\mathcal{M} = \left( \bigcap_{P \in \mathcal{I}_\Delta \cup \mathcal{I}_{\Delta_0}} P \right) \cap \left( \bigcap_{P \in \mathcal{I}_{\Delta_00}} P \right).$$

In the above, if $\mathcal{I}_{\Delta_00}$ is empty then $\bigcap_{P \in \mathcal{I}_{\Delta_00}} P$ is taken to be $L^1(G, B)$. Since $\mathcal{I}_{\Delta_00}$ is utmost a finite set, and $M_{\gamma, \phi}$ is a prime ideal for each $\phi \in \Delta(B)$, $\bigcap_{P \in \mathcal{I}_{\Delta_00}} P \nsubseteq M_{\gamma, \phi}$. Let $f \in \bigcap_{P \in \mathcal{I}_\Delta \cup \mathcal{I}_{\Delta_0}} P$. Choose $g \in (\bigcap_{P \in \mathcal{I}_{\Delta_00}} P \setminus M_{\gamma, \phi})$. Then $fg \in M_\gamma$. Since $\phi(g(\gamma)) \neq 0$ for each $\phi \in \Delta(B)$, by the assumption on $B$, $\hat{f}(\gamma)$ belongs to every minimal prime ideal of $B$. Since $B$ is semiprime, $\hat{f}(\gamma) = \theta$. Thus $M_\gamma = \bigcap_{P \in \mathcal{I}_\Delta \cup \mathcal{I}_{\Delta_0}} P$. This implies $\exists \subseteq M_\gamma$. This completes the proof of the claim.

For the remainder of the proof, the argument is similar to Theorem 3.3 of [7].

Let $\Gamma_{\Delta} = \{ \gamma_1, \gamma_2, \cdots, \gamma_n \}$. Let $h \in (G \cap (\cap_{i=2}^n M_{\gamma_i} )) \setminus M_{\gamma_1}$. Since there exists a minimal prime ideal $P \in \mathcal{M}$ contains $M_{\gamma_1}$ but not any of the $M_{\gamma_i}$'s for $2 \leq i \leq n$, such a function $h$ exists. Since $\hat{h}(\gamma_1) \neq \theta$, there exists a continuous linear functional $\lambda$ on $B$ such that $\lambda(\hat{f}(\gamma_1)) \neq 0$. Consider the basic open set

$$N = \{ \gamma \in \Gamma : |\lambda(\hat{h}(\gamma)) - \lambda(\hat{h}(\gamma_1))| < |\lambda(\hat{h}(\gamma_1))| \}.$$
of $\Gamma$ containing $\gamma_1$. Since $G$ is a noncompact Abelian group, $\gamma_1$ is not an isolated point in $\Gamma$. By the choice of $h$, the characters $\gamma_2, \gamma_3, \cdots, \gamma_n$ do not belong to $N$. Hence there exists a character $\gamma_0 \in \Gamma \setminus \{\gamma_1, \gamma_2, \cdots, \gamma_n\}$ such that $\gamma_0 \in N$. Since $\Theta$ is contained in $M_{\gamma_0}, \hat{h}(\gamma_0) = \theta$. Hence $|\lambda(\hat{h}(\gamma_1))| = |\lambda(\hat{h}(\gamma_1)) - \lambda(\hat{h}(\gamma_0))| < |\lambda(\hat{h}(\gamma_1))|$. This is a contradiction. Therefore $L^1(G, B)$ does not contain a non-trivial separating ideal.

The following result extends Theorem 3.3 of [7] (which in turn extends Theorem 5 of [11]) to some semiprime Banach algebras which do not posses the multiplicative identity.

Theorem 4.3. Let $G$ be a noncompact locally compact Abelian group, and $B$ be a commutative semiprime Banach algebra in which every minimal prime ideal is contained in a regular maximal ideal. Then every derivation on $L^1(G, B)$ is continuous. Also every epimorphism form a commutative Banach algebra onto $L^1(G, B)$ is continuous.

Proof. Obviously follows from Theorem 4.1 and the closed graph theorem.

Remark. If $B$ has the multiplicative identity then every proper prime ideal is contained in a maximal ideal of $B$. Even if $B$ does not have the multiplicative identity, in most of the algebras every minimal prime ideal is contained in a regular maximal ideal. Therefore the assumption in the above theorem that every minimal prime ideal contained in a regular maximal ideal of the algebra is not too restrictive.
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