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ON THE SEPARATING IDEALS OF SOME VECTOR-VALUED GROUP ALGEBRAS

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Abstract. For a locally compact Abelian group G, and a commutative Banach algebra B, let $L^1(G, B)$ be the Banach algebra of all Bochner integrable functions. We show that if G is noncompact and B is a semiprime Banach algebras in which every minimal prime ideal is contained in a regular maximal ideal, then $L^1(G, B)$ contains no nontrivial separating ideal. As a consequence we deduce some automatic continuity results for $L^1(G, B)$.

1. INTRODUCTION. For any locally compact Abelian group G, and commutative Banach algebra B, let $L^1(G, B)$ denote the convolution algebra of all integrable functions on G with values in B. As one might expect, there are some interesting similarities between B and $L^1(G, B)$. For instance, $L^1(G, B)$ is semi-simple if and only if B is semi-simple, and the regular maximal ideals of $L^1(G, B)$ are closely related in a natural way with the regular maximal ideals of both $L^1(G, B)$ and B. Also, $L^1(G, B)$ is Tauberian if and only if B is Tauberian. Refer to [8,9] for the proofs of the above results. Also it is easy to note that $L^1(G, B)$ is semiprime when B is semiprime. The question whether the zero ideal is the only separating ideal in a semiprime Banach algebra still seems to be open. However, in this paper we prove that when G is a noncompact locally compact Abelian group, and B is a commutative semiprime Banach algebra (not necessarily unital) in which every minimal prime ideal is contained in a regular maximal ideal, then $L^1(G, B)$ contains no non-trivial separating ideal. As a consequence we deduce some automatic continuity results for the algebra $L^1(G, B)$. Our results extend some of the results in [11] for non unital Banach algebras, and also extend some results in [7] for semiprime Banach algebras. For relevant information on $L^1(G, B)$ and for related results in harmonic analysis on Abelian groups,

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see [5,8,9,12].

2. PRELIMINARIES.

Let B be a commutative Banach algebra (not necessarily unital), and let G be a locally compact Abelian group with Haar measure m. Throughout the following, the dual group of G is denoted by Γ and the spectrum of B is denoted by $\Delta(B)$. Let $L^1(G,B)$ denote the Banach algebra of all integrable function from G into B,

$$(f*g)(t) := \int_G f(t-s)g(s)dm \text{ for all } f,g \in L^1(G,B) \text{ and } t \in G,$$

and let $\|f\|_1 := \int_G \|f(t)\|dm(t)$ for all $f \in L^1(G,B)$. Recall that for any $f \in L^1(G,B)$, and γ in the dual group Γ of G, $\hat{f}(\gamma) = \int_G \overline{\gamma(t)} f(t)dm(t)$ is known as the vector-valued Fourier transform of f at γ. Furthermore for any $\gamma \in \Gamma$, let $M_\gamma := \{f \in L^1(G,B) : \hat{f}(\gamma) = \theta\}$ where θ is the zero vector of B. Clearly, M_γ is a closed ideal of $L^1(G,B)$. If B has no non-trivial zero divisors, then M_γ is a closed prime ideal of $L^1(G,B)$. Recall that an ideal I of a commutative Banach algebra is said to be prime if the product $xy \in I$ only if either $x \in I$ or $y \in I$. It is an easy consequence of the Hahn-Banach theorem that $\bigcap_{\gamma\in\Gamma} M_\gamma$ is the zero ideal in $L^1(G,B)$. For any $\gamma \in \Gamma$, $\phi \in \Delta(B)$, let

$$M_{\gamma,\phi} := \{f \in L^1(G,B) | \phi(\hat{f}(\gamma)) = 0\}.$$

The regular maximal ideals of $L^1(G,B)$ are given by $M_{\gamma,\phi}$ for some $\gamma \in \Gamma$, and $\phi \in \Delta(B)$ ([8]).

For each $f \in L^1(G)$, and $x \in B$, we let

$$(f \otimes x)(s) = f(s)x \text{ for all } s \in G.$$

We recall some of the properties of the product $f \otimes x$ in the following proposition.

Proposition 2.1. Let G be a locally compact Abelian group, and let B be a commutative Banach algebra. Let $x, y \in B$; $f, g \in L^1(G)$; and γ a non-trivial continuous character on G. Then,

(i) $f \otimes x \in L^1(G,B)$, and $\|f \otimes x\|_1 = \|f\|_1 \|x\|$

(ii) $(f \pm g) \otimes x = f \otimes x \pm g \otimes x$

(iii) $\widehat{f \otimes x}(\gamma) = \hat{f}(\gamma)x$

(iv) $(f \otimes x) * (g \otimes x) = (f * g) \otimes xy$

(v) If B has the multiplicative identity 1, then $(f*g)\otimes x = (f\otimes x)*(g\otimes 1) = (f\otimes 1)*(g\otimes x)$

(vi) If $f_n \to f$ in $L^1(G)$ and $x_n \to x$ in B, then $f_n \otimes x_n \to f \otimes x$ in $L^1(G,B)$.

3. Main Results

Before we get to the main results, we need the following lemmas.

Lemma 3.1. Let G be a noncompact locally compact Abelian group, B a commutative Banach algebra, and f a non-zero function in $L^1(G,B)$. For a given γ in the dual group Γ of G and a positive number ε, there exist $f_1, f_2, \ldots, f_n$ in $L^1(G)$ with compactly supported Fourier transforms and $x_1, x_2, \ldots, x_n$ in B such that $\|f - \sum_{i=1} f_i \otimes x_i\| < \epsilon + \|\hat{f}(\gamma)\|$, where $\hat{f}_i(\gamma) = 0$ for $1 \le i \le n$.

Proof. Since finite linear combinations of the elements of the form $h \otimes x$ where $h \in L^1(G)$, and $x \in B$ are dense in $L^1(G)$, and the functions in $L^1(G)$ with compactly supported Fourier transforms are dense in $L^1(G)$, there exist $h_1, h_2, \ldots, h_n$ in $L^1(G)$ with compactly supported Fourier transforms and $x_1, x_2, \ldots, x_n$ in B such that $\|f - \sum_{i=1} h_i \otimes x_i\| < \frac{\varepsilon}{2}$. For $1 \le i \le n$, let $Supp\ \hat{h}_i = \{\alpha \in \Gamma : \hat{h}_i(\alpha) \ne 0\}$. For each $1 \le i \le n$, we define

$$g(t) = \frac{\chi_{(\cup_{j=1}^n Supp\ \hat{h}_j)}}{m(\cup_{j=1}^n Supp\ \hat{h}_i)} \gamma(t),$$

where $\chi_{(\cup_{j=1}^n Supp\ \hat{h}_j)}$ is the characteristic function of $(\cup_{j=1}^n Supp\ \hat{h}_i)$, and $f_i = h_i - \hat{h}_i(\gamma)g$. Clearly g and the f_i's belong to $L^1(G)$. It is easy to see that $\hat{g}(\gamma) = 1$, $\hat{f}_i(\gamma) = 0$ for each

i, and $\|g\|_1 = 1$. We have

$$\begin{aligned}
&\|f - \sum_{i=1} (f_i \otimes x_i) - g \otimes \hat{f}(\gamma)\| \\
&= \|f - \sum_{i=1} (h_i \otimes x_i) + \sum_{i=1} (h_i \otimes x_i) - \sum_{i=1} (f_i \otimes x_i) - g \otimes \hat{f}(\gamma)\| \\
&\leq \|f - \sum_{i=1} (h_i \otimes x_i)\| + \|\sum_{i=1} (h_i - f_i) \otimes x_i - g \otimes \hat{f}(\gamma)\| \quad \ldots (A)
\end{aligned}$$

Furthermore,

$$\begin{aligned}
&\|\sum_{i=1} (h_i - f_i) \otimes x_i - g \otimes \hat{f}(\gamma)| = \|\sum_{i=1} \hat{h}_i(\gamma) g \otimes s_i - g \otimes \hat{f}(\gamma)\| \\
&= \int_G \|\sum_{i=1} \hat{h}_i(\gamma) g(t) x_i - g(t)\hat{f}(\gamma)\| dm(t) \\
&= \frac{1}{m(\cup_{j=1}^n Supp\ \hat{h}_j)} \int_G \|\sum_{i=1} \hat{h}_i(\gamma) \chi_{(\cup_{j=1}^n Supp\ \hat{h}_j)} \gamma(t) x_i - \chi_{(\cup_{j=1}^n Supp\ \hat{h}_j)} \gamma(t) \hat{f}(\gamma)\| dm(t) \\
&= \frac{1}{m(\cup_{j=1}^n Supp\ \hat{h}_j)} \int_{(\cup_{j=1}^n Supp\ h_j)} \|\sum_{i=1} \hat{h}_i(\gamma) x_i - \hat{f}(\gamma)\| dm(t) \\
&= \|\sum_{i=1} \hat{h}_i(\gamma) x_i - \hat{f}(\gamma)\| \leq \|f - \sum_{i=1} h_i \otimes x_i\| < \frac{\varepsilon}{2} \quad \ldots (B)
\end{aligned}$$

From (A) and (B) it follows that $\|F - \sum_{i=1} f_i \otimes x_i - g \otimes \hat{f}(\gamma)\| < \epsilon$. Hence $\|f - \sum_{i=1} f_i \otimes x_i\| < \epsilon + \|\hat{f}(\gamma)\|$. This completes the proof of the Lemma. $\square$

Lemma 3.2. Let G be a noncompact locally compact Abelian group, B a commutative Banach algebra, and f a non-zero function in $L^1(G, B)$. For a given γ in the dual group Γ of G and a given positive number $\epsilon > 0$, there exist $g_1, g_2, \ldots, g_n$ in $L^1(G)$, a neighborhood V of γ, and $x_1, x_2, \ldots, x_n$ in B such that

$$\|f - \sum_{i=1} g_i \otimes x_i\| < \epsilon + \|\hat{f}(\gamma)\|$$

where $\hat{g}_i = 0$ on V for $1 \leq i \leq n$.

Proof. By Lemma 3.1, there exist $f_1, f_2, \dots, f_n$ in $L^1(G)$ with compactly supported Fourier transforms, and $x_1, x_2, \dots, x_n$ in B such that

$$\|f - \sum_{i=1}^{n} f_i \otimes x_i\| < \frac{\epsilon}{2} + \|\hat{f}(\gamma)\|$$

where $\hat{f}_i(\gamma) = 0$. Since $L^1(G)$ satisfies the Ditkin's condition ([12]), there exist $g_1, g_2, \dots, g_n$ in $L^1(G)$, and a neighborhood V of γ such that $\hat{g}_i = 0$ on V, and

$$\|f_i - g_i\|_1 < \frac{\epsilon}{2(1 + \sum_{i=1}^{n} \|x_i\|)}$$

for $1 \le i \le n$. Now

$$\begin{aligned}
\|f - \sum_{i=1}^{n} g_i \otimes x_i\|_1 &\le \|f - \sum_{i=1}^{n} f_i \otimes x_i\|_1 + \|\sum_{i=1}^{n} (f_i - g_i) \otimes x_i\|_1 \\
&\le \frac{\epsilon}{2} + \|\hat{f}(\gamma)\| + \sum_{i=1}^{n} \|f_i - g_i\|_1 \|x_i\| \\
&< \frac{\epsilon}{2} + \|\hat{f}(\gamma)\| + \frac{\epsilon}{2(1 + \sum_{i=1}^{n} \|x_i\|)} \left(\sum_{i=1}^{n} \|x_i\|\right) \\
&= \epsilon + \|\hat{f}(\gamma)\|. \qquad \square
\end{aligned}$$

Corollary 3.3. Let $f \in L^1(G, B)$, and $\gamma \in \Gamma$ such that $\hat{f}(\gamma) = \theta$. Given $\epsilon > 0$, there exist $g_1, g_2, \dots g_n$ in $L^1(G)$ with a vanishing Fourier transform in a neighborhood V of γ, and $x_1, x_2, \dots, x_n$ in B such that $\|f - \sum_{i=1}^{n} g_i \otimes x_i\| < \epsilon$.

Proof. Obviously follows from the Lemma 3.2. $\square$

Now we are ready for the main results of the section.

Theorem 3.4 Let G be a locally compact Abelian group, γ a continuous character on G, and $\mathcal{P}$ a prime ideal contained in M_γ. Then $\mathcal{P}$ is dense in M_γ.

Proof. Let $\mathcal{P}$ be a prime ideal of $L^1(G, B)$ contained in M_γ. Let f be a function with $\hat{f}$ identically equal to the zero vector in a neighborhood V of γ. We claim that f belongs to $\mathcal{P}$. For, if g belongs to $L^1(G)$ with $\hat{g}(\gamma) \neq 0$, $\hat{g} = 0$ on $\Gamma - V$, and x a non-zero vector in B, then $(g \otimes x) * f = \Theta$ (the zero vector of $L^1(G, B)$). Since $\mathcal{P}$ is a prime ideal of $L^1(G, B)$, either $g \otimes x \in \mathcal{P}$ or $f \in \mathcal{P}$. But $g \hat{\otimes} x(\gamma) = \hat{g}(\gamma)x \neq \theta$. Hence $f \in \mathcal{P}$. Thus all the functions f in $L^1(G, B)$ with vanishing Fourier transforms in a neighborhood of γ belong to $\mathcal{P}$. Hence by Lemma 3.2, it follows that $\mathcal{P}$ is dense in M_γ. This completes the proof of the theorem. ¥

Theorem 3.5. Let G be a noncompact locally compact Abelian group, and B be a commutative Banach algebra. If $\mathcal{P}$ is a closed prime ideal of $L^1(G, B)$ contained in $M_{\gamma,\phi}$ for some $\gamma \in \Gamma$, and $\phi \in \Delta(B)$, then $\mathcal{P}$ contains M_γ. Furthermore $\mathcal{P}$ does not contain M_σ for any $\sigma \neq \gamma$.

Proof. Let $f \in M_\gamma$. By Corollary 3.3, f can be approximated by a function g in $L^1(G, B)$ with vanishing Fourier transform in a neighborhood V of γ. By an argument similar to the one given in Theorem 3.4, we can show $g \in \mathcal{P}$. Since $\mathcal{P}$ is a closed ideal, it follows that $f \in \mathcal{P}$. Thus M_γ is contained in $\mathcal{P}$. Let $\sigma \in \Gamma$ such that $\sigma \neq \gamma$. Suppose V_σ and V_γ are compact neighborhoods of σ and γ respectively such $V_\sigma \cap V_\gamma = \emptyset$. Then there exist functions f_σ and f_γ from G into the complex plane with the support of $\hat{f}_\sigma$ contained in V_σ and the support of $\hat{f}_\gamma$ contained in V_γ such that $\hat{f}_\sigma(\sigma) = 1$ and $\hat{f}_\gamma(\gamma) = 1$. Let $x, y \in B$ such that $\phi(x)\phi(y) \neq 0$. Then $f_\sigma \otimes x,\ f_\gamma \otimes y \in L^1(G, B)$ such that $(f_\sigma \otimes x) * (f_\sigma \otimes y) = \Theta$. Since $\mathcal{P}$ is a prime ideal contained in $M_{\gamma,\sigma}$, we get $f_\sigma \otimes x \in \mathcal{P}$. Obviously $f_\gamma \otimes y \notin \mathcal{P}$. However $f_\gamma \otimes y \in M_\sigma$. Therefore M_σ is not contained in $\mathcal{P}$. ¥

4. Applications.

Recall that a closed ideal S of a commutative Banach algebra A is called a separating ideal ([3]) if it satisfies the following condition: For each sequence $\{a_k\}_{k\geq 1}$ in A there is a positive integer n such that $\overline{a_1 a_2 \cdots a_n S} = \overline{a_1 a_2 \cdots a_k S}$ $(k \geq n)$. For any derivation D on A, let $\Im(D) =: \{a \in A |$ there is a sequence $\{a_n\}$ in A with $a_n \to 0$ and $Da_n \to a\}$. For any epimorphism h form a commutative Banach algebra X onto A, let $\Im(h) =: \{a \in A|$ there is a sequence $\{x_n\}$ in X with $x_n \to 0$ and $h(x_n) \to a\}$. It is easy to show that $\Im(D)$,and $\Im(h)$ are closed ideals of A. By the closed graph theorem D is continuous if and only if $\Im(D)$ is zero. Similarly h is continuous if and only if $\Im(h)$ is zero. It is well known that $\Im(D)$ and $\Im(h)$ are separating ideals of A ([13]). For further information on separating ideals, their relation to the prime ideals of the Banach algebra, and for related results on automatic continuity theory, see [1,2,3,4,6,10].

Now we are ready to state one of the main results of the section.

Theorem 4.1. Let G be a noncompact locally compact Abelian group G, and B a commutative semiprime Banach algebra in which every minimal prime ideal is contained in a regular maximal ideal. Then $L^1(G, B)$ contains no nontrivial separating ideal.

Lemma 4.2. Let G be a noncompact locally compact Abelian group G, and B a commutative semiprime Banach algebra. For any $\gamma \in \Gamma$, $M_\gamma = \bigcap_{\mathcal{P} \in \mathcal{I}_\gamma} \mathcal{P}$ where $\mathcal{I}_\gamma$ is the set of all minimal prime ideals of $L^1(G, B)$ containing M_γ.

Proof. Let $f \in \bigcap_{\mathcal{P} \in \mathcal{I}_\gamma} \mathcal{P}$. Since there is a one-to-one correspondence between the prime ideals of the quotient algebra $L^1(G, B)/M_\gamma$ and the prime ideals of the algebra $L^1(G, B)$ containing M_γ, there exists a positive integer n such that $\underbrace{f * f * \cdots * f}_{n \text{ times}} \in M_\gamma$. This implies $(\hat{f}(\gamma))^n = \theta$. Since B is semiprime, $\hat{f}(\gamma) = \theta$. Hence $f \in M_\gamma$. ¥

Proof of Theorem 4.1. If possible assume that $\Im$ is a nontrivial separating ideal in $L^1(G, B)$.

Claim. $\Im$ is contained in all but finitely many M_γ for $\gamma \in \Gamma$.

Proof of the claim. Let $\mathcal{M}$ be the set of all minimal prime ideals of $L^1(G, B)$ not containing $\Im$. By [3] $\mathcal{M}$ is a finite set. Let

$$\mathcal{M}_\Delta = \{\mathcal{P} \in \mathcal{M} | \mathcal{P} \subseteq M_{\gamma,\phi} \text{ for some } (\gamma, \phi) \in \Gamma \times \Delta(B)\}$$

and $\mathcal{M}_{\Delta^0} = \mathcal{M} - \mathcal{M}_\Delta$. By Theorem 3.5, each member of $\mathcal{M}_\Delta$ contains a unique M_γ for some $\gamma \in \Gamma$. Let $\Gamma_{\mathcal{M}_\Delta} = \{\gamma \in \Gamma | M_\gamma \subseteq \mathcal{P}$ for some $\mathcal{P} \in \mathcal{M}_\Delta\}$. Obviously $\Gamma_{\mathcal{M}_\Delta}$ is a finite set. Since $\Im$ is contained in all but finitely many closed prime ideals of $L^1(G, B)$ ([3]), and since any prime ideal contains a minimal prime ideal, it follows that $\Gamma_{\mathcal{M}_\Delta}$ is not empty. Let $\gamma \in \Gamma - \Gamma_{\mathcal{M}_\Delta}$. By Lemma 4.2, $M_\gamma = \bigcap_{\mathcal{P} \in \mathcal{I}_\gamma} \mathcal{P}$ where $\mathcal{I}_\gamma$ is the set consisting of all minimal prime ideals of $L^1(G, B)$ containing M_γ. Write $\mathcal{I}_\gamma = \mathcal{I}_\Delta \cup \mathcal{I}_{\Delta^0} \cup \mathcal{I}_{\Delta^{00}}$ where

$$\mathcal{I}_\Delta = \{\mathcal{P} \in \mathcal{I}_\gamma | \mathcal{P} \subseteq M_{\gamma,\phi} \text{ for some } \phi \in \Delta(B)\},$$

$$\mathcal{I}_{\Delta^0} = \{\mathcal{P} \in \mathcal{I}_\gamma | \mathcal{P} \text{ contains } \Im, \text{ and } \mathcal{P} \not\subseteq M_{\gamma,\phi} \text{ for each } \phi \in \Delta(B)\}$$

and

$$\mathcal{I}_{\Delta^{00}} = \{\mathcal{P} \in \mathcal{I}_\gamma | \mathcal{P} \text{ does not contain } \Im \text{ and } \mathcal{P} \not\subseteq M_{\gamma,\phi} \text{ for each } \phi \in \Delta(B)\}.$$

Notice that $\mathcal{I}_{\Delta^{00}}$ is almost a finite set, and each $\mathcal{P}$ in $\mathcal{I}_\Delta$ contains $\Im$. Obviously

$$\mathcal{M} = (\bigcap_{\mathcal{P} \in \mathcal{I}_\Delta \cup \mathcal{I}_{\Delta^0}} \mathcal{P}) \cap (\bigcap_{\mathcal{P} \in \mathcal{I}_{\Delta^{00}}} \mathcal{P}).$$

In the above, if $\mathcal{I}_{\Delta^{00}}$ is empty then $\bigcap_{\mathcal{P} \in \mathcal{I}_{\Delta^{00}}} \mathcal{P}$ is taken to be $L^1(G, B)$. Since $\mathcal{I}_{\Delta^{00}}$ is utmost a finite set, and $M_{\gamma,\phi}$ is a prime ideal for each $\phi \in \Delta(B)$, $\bigcap_{\mathcal{P} \in \mathcal{I}_{\Delta^{00}}} \mathcal{P} \not\subseteq M_{\gamma,\phi}$. Let $f \in \bigcap_{\mathcal{P} \in \mathcal{I}_\Delta \cup \mathcal{I}_{\Delta^0}} \mathcal{P}$. Choose $g \in (\bigcap_{\mathcal{P} \in \mathcal{I}_{\Delta^{00}}} \mathcal{P} \backslash M_{\gamma,\phi})$. Then $fg \in M_\gamma$. Since $\phi(\hat{g}(\gamma)) \neq 0$ for each $\phi \in \Delta(B)$, by the assumption on B, $\hat{f}(\gamma)$ belongs to every minimal prime ideal of B. Since B is semiprime, $\hat{f}(\gamma) = \theta$. Thus $M_\gamma = \bigcap_{\mathcal{P} \in \mathcal{I}_\Delta \cup \mathcal{I}_{\Delta^0}} \mathcal{P}$. This implies $\Im \subset M_\gamma$. This completes the proof of the claim.

For the remainder of the proof, the argument is similar to Theorem 3.3 of [7].

Let $\Gamma_{\mathcal{M}_\Delta} = \{\gamma_1, \gamma_2, \cdots, \gamma_n\}$. Let $h \in (G \cap (\cap_{i=2}^n M_{\gamma_i})) \backslash M_{\gamma_1}$. Since there exists a minimal prime ideal $\mathcal{P} \in \mathcal{M}$ contains M_{γ_1} but not any of the M_{γ_i}'s for $2 \leq i \leq n$, such a function h exists. Since $\hat{h}(\gamma_1) \neq \theta$, there exists a continuous linear functional λ on B such that $\lambda(\hat{f}(\gamma_1)) \neq 0$. Consider the basic open set

$$N = \{\gamma \in \Gamma : |\lambda(\hat{h}(\gamma)) - \lambda(\hat{h}(\gamma_1))| < |\lambda(\hat{h}(\gamma_1))|\}$$

of Γ containing γ_1. Since G is a noncompact Abelian group, γ_1 is not an isolated point in Γ. By the choice of h, the characters $\gamma_2, \gamma_3, \cdots, \gamma_n$ do not belong to N. Hence there exists a character $\gamma_0 \in \Gamma \backslash \{\gamma_1, \gamma_2, \cdots, \gamma_n\}$ such that $\gamma_0 \in N$. Since $\Im$ is contained in M_{γ_0}, $\hat{h}(\gamma_0) = \theta$. Hence $|\lambda(\hat{h}(\gamma_1))| = |\lambda(\hat{h}(\gamma_1)) - \lambda(\hat{h}(\gamma_0))| < |\lambda(\hat{h}(\gamma_1))|$. This is a contradiction. Therefore $L^1(G, B)$ does not contain a non-trivial separating ideal. ¥

The following result extends Theorem 3.3 of [7] (which in turn extends Theorem 5 of [11]) to some semiprime Banach algebras which do not posses the multiplicative identity.

Theorem 4.3. Let G be a noncompact locally compact Abelian group, and B be a commutative semiprime Banach algebra in which every minimal prime ideal is contained in a regular maximal ideal. Then every derivation on $L^1(G, B)$ is continuous. Also every epimorphism form a commutative Banach algebra onto $L^1(G, B)$ is continuous.

Proof. Obviously follows from Theorem 4.1 and the closed graph theorem. ¥

Remark. If B has the multiplicative identity then every proper prime ideal is contained in a maximal ideal of B. Even if B does not have the multiplicative identity, in most of the algebras every minimal prime ideal is contained in a regular maximal ideal. Therefore the assumption in the above theorem that every minimal prime ideal contained in a regular maximal ideal of the algebra is not too restrictive.

REFERENCES

1. W.G. Bade and P.C. Curtis, Jr. Prime ideals and automatic continuity for Banach algebras. J. Funct. Anal., vol. 29, 1978, pp 88-103.
2. P.C. Curtis Jr. Derivations on commutative Banach algebras. in Proceedings, Long Beach, 1981, Lecture Notes in Math. (Springer-Verlag, Berlin, Heidelberg, New York), 975, 1983, pp 328-333.
3. J. Cusack. Automatic continuity and topologically simple radical Banach algebras. J. London Math. Soc., vol 16, 1977, pp 493-500.
4. H.G. Dales. Automatic continuity: A survey. Bull. London Math. Soc., vol 10, 1978, pp 129-183.
5. J. Diestel and J.J. Uhl. Vector measures. Math Surveys (Amer. Math. Soc., Providence, RI, vol 15, 1977.
6. R. Garimella. On nilpotency of the separating ideal of a derivation. Proc. Amer. Math. Soc., vol 117, 1993, pp 167-174.
7. R. Garimella. On continuity of derivations and epimorphisms on some vector-valued group algebras. Bull. Austral. Math. Soc., vol 56, 1997, pp 209-215.
8. A. Hausner. The Tauberian Theorem for Group algebras of vector-valued functions. Pacific J. Math., vol 7, 1957, pp 1603-1610.
9. G.P. Johnson. Space of function with values in a Banach algebra. Trans. Amer. Math. Soc., vol 92, 1959, pp 411-429.
10. M.M. Neumann.Automatic continuity of linear operations in Functional analysis, surveys and recent results II, North-Holland math. Studies (North-Holland, Amsterdam, New York), vol 38, 1980, pp 269-296.
11. M.M. Neumann and M.V. Velasco. Continuity of epimorphisms and derivations on vector-valued group algebras. Arch. Math. (Basel), vol 68, 1997, pp no.2 151-158.

12. W. Rudin. Fourier analysis on groups. Interscience, New York, London, 1962.

13. A.M. Sinclair. Automatic continuity of linear operators. London Mathematical Society Lecture Notes Series, Cambridge Univ. Press, London/New York, vol 21, 1976.

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