Lecture 1: INTRODUCTION TO CLIFFORD ALGEBRA

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ABSTRACT Text of the abstract here
Keywords: Clifford algebras, bilinear form, quadratic form.

1   Directions to the Presenters

This is a brief guide and a template for lecturers who will be presenting the "Lecture Series on Clifford Algebras and Applications, May 18 and 19, 2002" prior to the 6th Conference on Clifford Algebras.

The six lectures are intended to prepare graduate students, post-docs, and all other persons who have not worked in the area of Clifford algebras, for the actual talks that will be presented at the 6th Conference. Therefore, it is very important that these lectures be:

- Very well cross-referenced [this will require close collaboration among the six lecturers],

- Very carefully written in terms of pedagogy and contents, i.e., there should be no narrow foci on one's own work,

- Definitions should be adopted and used throughout, no repetitions of definitions should be taking place, notation should be the same. Please use Clifford algebra macros from `Macro98.tex` file,

- Lectures should accessible to graduate students: we want to be able to sell this book as a textbook for some graduate course, and we would like it to be a good book,

- It would be useful and expected from a textbook to have at the end of each lecture a list of problems for students to solve. Problems should

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range from easy to hard. See for example [3] where such problems appear.

- There should be an index: please insert `\index` commands where appropriate. This command is used as follows:

```
\index{Yang-Baxter equation!quantum}%
\index{quantum!Yang-Baxter equation}%
```

- The editing of the material for each lecture should be done very carefully and thoroughly by the lecturer and is the responsibility of the lecturer. The editors of the series will do their best to ensure that the lectures fit together smoothly with no contradictory remarks or points of view. The lectures should represent modern and contemporary points of view. Finally, there should be no copyright problems.

1.1   Preamble

In the preamble, please use `\documentclass[twosided]{reporteq}...` environment along with `svcon2e.sty` style file. You are welcome to use the following packages/style files from $\mathcal{AMS}$-LaTeX:

```
\documentclass[twoside]{reporteq}
\usepackage{svcon2e}
\usepackage{makeidx}
\usepackage{latexsym}
\usepackage{amsmath}
\usepackage{amsfonts}
\usepackage{amssymb}
\usepackage{graphicx}
\usepackage{amsthm}
\input{Makro98}
\input{psfig}
\mathsurround 1.5pt
\renewcommand{\indexspace}{\vskip 0.5ex} %sets correct spacing between index items
\makeindex
\renewcommand{\theequation}{\arabic{section}.\arabic{equation}}}
```

1.2   Common environments

To assure uniformity of styles, to minimize editing, and to speed up manuscript preparation, please use newer environments as much as possible. These new invironments are:

```
\begin{align}...\end{align}
\begin{align*}...\end{align*}
```

```
\begin{gather}...\end{gather}
\begin{gather*}...\end{gather*}
\begin{multline}...\end{multline}
\begin{gather*}...\end{gather*}
\begin{gather*}...\end{gather*}
```

Environment \begin{align}...\end{align} is much more convenient to use than \begin{eqnarray}...\end{egnarray}
Here is a sample of these new environments and their usage:

```
\begin{align}
t_i^2 &= (1-q) t_i + q,                          \label{eq: t1} \\
t_i t_j &= t_j t_i,   \quad \vert i-j \vert \ge 2, \label{eq: t2} \\
t_i t_{i+1} t_i &= t_{i+1} t_i t_{i+1}.           \label{eq: t3}
\end{align}

\begin{align*}
R(12)_{q}R(12)_{q} &= \frac{(1 + q) \be_1 \wedge \be_5 + q(1 + q )}{(1 + q)^2}\\
                   &= \frac{\be_1 \wedge \be_5 +q}{1+q}= \frac{b_1 + q}{1+q} = R(12)_{q}.
\end{align*}

\begin{gather*}
b_1 = \be_1 \w \be_5, \; b_2 = \be_2 \w \be_6, \\
b_{12} = b_1 b_2 = -(1 + q) \Id + \be_1 \w \be_6 - \be_1 \w \be_2 \w \be_5 \w \be_6 +
                  (1 + q) \e_2 \w \be_5, \\
b_{21} = b_2 b_1 =-q(1+q)\Id + (1+q)\be_1 \w \be_6 - \be_1 \w \be_2 \w \be_5 \w \be_6 +
                  q\be_2 \w \be_5, \\
b_{121} = b_1 b_2 b_1 = q\be_1\w\be_5-(1+2q)\Id+q\be_2\w\be_6+\be_1\w\be_6 \\
                       +(-1+q)\be_1\w\be_2\w\be_5\w\be_6-(-q-1+q^2)\be_2\w\be_5.
\end{gather*}

\begin{multline}
(1+q)z^2+(-q^2 K_4+K_4+q K_2-1+K_2)z+K_4 K_2 + K_2^2\\
-K_4-K_2+q K_4-q^2 K_4^2-q K_4^2-q^2 K_2 K_4+q K_2^2 = 0,
\label{eq:alphaeq}
\end{multline}
```

1.3   Inserting figures

To insert figures, please use

```
\begin{figure}[ht]
\centerline{\psfig{figure=braid.eps,height=3.0cm,width=2.0cm}}
\caption{Tangles representing equation (\ref{eq:b3}).}
\label{fig:tangles}
\end{figure}
```

1.4   Bibliography

Please use the following style for the bibliography:

```
\begin{thebibliography}{99}
\def\topsep{0pt}
        \def\parsep{0pt plus 5pt minus 1pt}
        %\def\parsep{-4pt plus 5pt minus 0pt}
        \def\itemsep{-0.5ex}
        %\normalsize
        \small
        %\footnotesize
\bibitem{AblamowiczLounesto} R.~Ab\l amowicz, P. Lounesto,
On Clifford algebras of a bilinear form with an antisymmetric
part, in {\it Clifford Algebras with Numeric and Symbolic
Computations,\/} Eds. R. Ab\l amowicz, P. Lounesto, J. M. Parra,
Birkh\"auser, Boston, 1996, 167--188.
%
\bibitem{Ablamowicz} R.~Ab\l amowicz, Clifford algebra
computations with Maple, {\it Clifford (Geometric) Algebras,\/}
Banff, Alberta Canada, 1995, Ed. W. E. Baylis, Birkh\"auser,
Boston, 1996, 463--501.
%
\end{thebibliography}
```

1.5   Appendices

Before the appendices, please insert these commands:

```
\renewcommand{\thesection}{\Alph{section}}
\setcounter{section}{0}
\renewcommand{\theequation}{\Alph{section}.\arabic{equation}}
\section{Appendix}
```

that give new numbering for the equations in each appendix.

A   Appendix

This is an equation in the appendix

$$f(x) = \cos(x) \tag{A.1}$$