Instructions for Contributing Authors – Advanced Mathematical Tools: A Frontier Between Mathematics and Engineering 43rd Annual Technical Meeting Society of Engineering Science August 13-16th, 2006 PennState, University Park

Rafał Abłamowicz and Jane Liu

ABSTRACT This is a short guide for the Authors how to prepare manuscripts of papers that are to be presented at the Symposium "Advanced Mathematical Tools: A Frontier Between Mathematics and Engineering" during the 43rd Annual Technical Meeting of the Society of Engineering Science, August 13-16th, 2006, at PennState, University Park. Preamble commands, the style file, the macros, inserting of pictures, indexing, including appendices, and references, are all discussed in this guide.
Keywords: Clifford algebras, Groebner basis, finite element analysis

1 Directions for the Authors

This is a brief guide and a template for presenters who will be presenting papers at the Symposium "Advanced Mathematical Tools: A Frontier Between Mathematics and Engineering" during the 43rd Annual Technical Meeting of the Society of Engineering Science, August 13-16th, 2006, at PennState, University Park. In order to minimize time and effort needed to facilitate quick and efficient editing of post-conference volume contain-

AMS Subject Classification: 15A66, 17B37, 20C30, 81R25.

ing papers accepted for publication, Organizers of the Symposium request that all Authors use these guidelines when preparing their papers.

Here are some general points that we ask all Authors keep in mind when preparing their manuscripts:

- All contributions need to be typeset in $\mathcal{AMS}$-LaTeX using LaTeX2e and `svcon2e.sty` style file. Please use this document as a template. In particular, please provide a list of keywords and a listing of appropriate AMS Subject Classification numbers in the footnote (as shown in this paper). Please insert appropriate arguments in the `\markboth` command (see the source code of this paper as a sample).

- Definitions and notation should be, as much as it is possible, consistent from paper to paper. Therefore, Authors are asked to use Clifford algebra macros from the `Macro98.tex` file.

- Please use `\mathbb{K}`, `\mathbb{F}`, `\mathbb{C}`, `\mathbb{R}`, `\mathbb{H}`, etc. to produce field names $\mathbb{K}, \mathbb{F}, \mathbb{C}, \mathbb{R}, \mathbb{H}$. It is convenient to define the following new commands in the preamble:

```
\newcommand{\BK}{\mathbb{K}}
\newcommand{\BF}{\mathbb{F}}
\newcommand{\BC}{\mathbb{C}}
\newcommand{\BR}{\mathbb{R}}
\newcommand{\BH}{\mathbb{H}}
```

- All papers should be indexed: Please insert `\index` commands where appropriate. The `\index` command is used as follows:

```
\index{Yang-Baxter equation!quantum}
\index{quantum!Yang-Baxter equation}
```

  For more information how to construct the index, please typeset the enclosed file `makeindex.tex`. In order to typeset paper with index commands, the preamble needs to contain command `\makeindex` and file `makeindx.exe` needs to be available to the processor (it is best to put it together with the source file in the same directory).

- It is the responsibility of each Author to submit carefully typeset and edited material, and to make sure that there are no copyright problems of any kind. This will enable Editors of the post-conference volumes to ensure that the published papers fit together smoothly and use standard notation as much as possible.

1.1 Preamble

In the preamble, please use `\documentclass[twosided]{reporteq}...` environment along with `svcon2e.sty` style file. You are encouraged to use the following packages/style files from $\mathcal{AMS}$-LaTeX:

```
\documentclass[twoside]{reporteq}
\usepackage{svcon2e}
\usepackage{makeidx}
\usepackage{latexsym}
\usepackage{amsmath}
\usepackage{amsfonts}
\usepackage{amssymb}
\usepackage{graphicx}
\usepackage{amsthm}
\input{Makro98}
\input{psfig}
\mathsurround 1.5pt %sets extra spacing around inline math environment
\renewcommand{\indexspace}{\vskip 0.5ex} %sets spacing between index items
\makeindex %makes index files provided makeindx.exe is available
\renewcommand{\theequation}{\arabic{section}.\arabic{equation}}
```

Please notice in the above command

```
\renewcommand{\theequation}{\arabic{section}.\arabic{equation}}
```

which fixes numbering style throughout the paper. Furthermore, please observe that extra spacing around mathematical expressions in the text is afforded by

```
\mathsurround 1.5pt
```

For the `\mathsurround` command to work properly, one needs to put in commas, colons, periods, etc. inside of the math environments as in `$u \in \cl(B).$` or `$u \in \cl(B),$` to produce, $u \in C\ell(B)$. or $u \in C\ell(B)$, respectively. Placing commas, colons, periods, etc. outside of the math environment would produce incorrect spacing. For example, notice incorrect spacing of the final period in `$u \in \cl(B)$.` which gives $u \in C\ell(B)$. (this document has been processed with the `\mathsurround 1.5pt` command).

1.2 Clifford macros

Here is a sample of some text with Clifford macros in action taken from [4] (some references are missing, hence they produce this symbol [**?**] in the text below):

"Consider the reversion anti-automorphism of a Clifford algebra $C\ell(V,B)$ induced by a general, not necessarily symmetric bilinear form B. Let $\mathbf{e}_1,\mathbf{e}_2,\ldots,\mathbf{e}_n$ be a basis of V, and B_{ij} be the matrix of B in that basis. If we decompose B into its symmetric and antisymmetric parts $B=g+A$ [in characteristic $\neq 2$], then

$$\mathbf{e}_1\mathbf{e}_2=\mathbf{e}_1\wedge\mathbf{e}_2+B_{12}=\underbrace{\mathbf{e}_1\wedge\mathbf{e}_2+A_{12}}_{\mathbf{e}_1\,\dot\wedge\,\mathbf{e}_2}+g_{12} \tag{1.1}$$

and

$$\mathbf{e}_2\mathbf{e}_1=\mathbf{e}_2\wedge\mathbf{e}_1+B_{21}=\underbrace{-\mathbf{e}_1\wedge\mathbf{e}_2-A_{12}}_{-\mathbf{e}_1\,\dot\wedge\,\mathbf{e}_2}+g_{12} \tag{1.2}$$

where $B_{21}=-A_{12}+g_{12}$. Thus, since $\mathbf{e}_1\,\dot\wedge\,\mathbf{e}_2=\mathbf{e}_1\wedge\mathbf{e}_2+A_{12}\in K+\bigwedge^2 V$, the new exterior product $\dot\wedge$ does not give the same multivector structure as $\wedge$ unless B is symmetric." (see [4])

"This shows that reversion, unlike the grade involution, depends on the choice of the antisymmetric part of B, and cannot be defined, in the case of a non-symmetric B, in terms of the dimension grading as

$$\tilde u=\langle u\rangle_0+\langle u\rangle_1-\langle u\rangle_2-\langle u\rangle_3++--... \tag{1.3}$$

for $u=\langle u\rangle_0+\langle u\rangle_1+\langle u\rangle_2+\langle u\rangle_3+....$ with k-vector parts of u belonging to appropriate k-vector subspaces of $C\ell(Q)=K+V+\bigwedge^2 V+\ldots+\bigwedge^n V$.

Let us observe now how the contraction $\lrcorner$ and Clifford product in $C\ell(B)$ depend on B. In order to make that dependence explicit, from now on we write $\underset{B}{\lrcorner}$ instead of $\lrcorner$ and $\underset{B}{uv}$ instead of uv for any u and v in $C\ell(B)$. Notice first that from (**??**), which is of course true for any B,

$$\underset{B}{\mathbf{x}u}=\mathbf{x}\wedge u+\mathbf{x}\underset{B}{\lrcorner}u \tag{1}$$

(see also Lounesto 1995, page 138) one cannot conclude that

$$\underset{B}{\hat u\mathbf{x}}=\mathbf{x}\wedge u-\mathbf{x}\underset{B}{\lrcorner}u \tag{1.4}$$

in the case of a non-symmetric bilinear form B (see Equation (2) on page 139 of Lounesto 1995). Here $\hat u$ denotes grade involution [recall that grade involution is the unique automorphism of $C\ell(B)$ extending the mapping on V which changes signs of vectors.]" (see *ibid.*)

For more examples, please see recent publications [3] and [10].

Here is a brief legend with a sample usage:

- Commands `\cl(V,B)`,`\cl(Q)`,`\cl_{p,q}`,`\cl_{p,q,d}` give, respectively, these Clifford algebras: $C\ell(V,B)$, $C\ell(Q)$, $C\ell_{p,q}$, and $C\ell_{p,q,d}$.

- Commands `\Spin(3)` and `\Pin(3)` produce the spin group **Spin**(3) and the pin group **Pin**(3).

- Commands `$\hat u$`, `$\tilde u$`, and `$\bar u$` give, respectively, grade involution $\hat{u}$, reversion $\tilde{u}$, and conjugation $\bar{u}$ of $u \in C\ell(B)$.

- Undotted and dotted wedge operations are accomplished with macros `\w` and `\dw`, respectively, as in `$u \w v$` and `$u \dw v$` which give $u \wedge v$ and $u \dot\wedge v$. Furthermore, command `\bigw(V)` produces Grassmann algebra $\bigwedge(V)$.

- A left contraction $\mathbf{x} \lrcorner_B u$ and a right $u \llcorner_B \mathbf{x}$ with respect to a bilinear form B in $C\ell(V, B)$ is accomplished with these macros:

  ```
  $\bx\mathop{\JJ}\displaylimits_B u$
  ```

  and

  ```
  $u \mathop{\LL}\displaylimits_B \bx.$
  ```

To see other macro commands, please open file `Makro98.tex`.

1.3 Common environments

To assure uniformity of styles, to minimize editing, and to speed up manuscript preparation, please use newer environments as much as possible. These new environments are:

```
\begin{align}...\end{align}
\begin{align*}...\end{align*}
\begin{gather}...\end{gather}
\begin{gather*}...\end{gather*}
\begin{multline}...\end{multline}
\begin{gather*}...\end{gather*}
\begin{gather*}...\end{gather*}
```

Environment `\begin{align}...\end{align}` is much more convenient to use than `\begin{eqnarray}...\end{egnarray}`. Here is a sample of these new environments and their usage:

```
\begin{align}
t_i^2 &= (1-q) t_i + q,                              \label{eq: t1} \\
t_i t_j &= t_j t_i,   \quad \vert i-j \vert \ge 2, \label{eq: t2} \\
t_i t_{i+1} t_i &= t_{i+1} t_i t_{i+1}.             \label{eq: t3}
\end{align}
```

```
\begin{align*}
R(12)_{q}R(12)_{q} &= \frac{(1 + q) \be_1 \w \be_5 + q(1 + q )}{(1 + q)^2}\\
                   &= \frac{\be_1 \w \be_5 +q}{1+q}= \frac{b_1 + q}{1+q} = R(12)_{q}.
\end{align*}

\begin{gather*}
b_1 = \be_1 \w \be_5, \; b_2 = \be_2 \w \be_6, \\
b_{12} = b_1 b_2 = -(1 + q) \Id + \be_1 \w \be_6 - \be_1 \w \be_2 \w \be_5 \w \be_6 +
                    (1 + q) \e_2 \w \be_5, \\
b_{21} = b_2 b_1 =-q(1+q)\Id + (1+q)\be_1 \w \be_6 - \be_1 \w \be_2 \w \be_5 \w \be_6 -
                   q\be_2 \w \be_5, \\
b_{121} = b_1 b_2 b_1 = q\be_1\w\be_5-(1+2q)\Id+q\be_2\w\be_6+\be_1\w\be_6 \\
                        +(-1+q)\be_1\w\be_2\w\be_5\w\be_6-(-q-1+q^2)\be_2\w\be_5.
\end{gather*}

\begin{multline}
(1+q)z^2+(-q^2 K_4+K_4+q K_2-1+K_2)z+K_4 K_2 + K_2^2\\
-K_4-K_2+q K_4-q^2 K_4^2-q K_4^2-q^2 K_2 K_4+q K_2^2 = 0,
\label{eq:alphaeq}
\end{multline}
```

1.4 Inserting figures

To insert figures, please use

```
\begin{figure}[ht]
\centerline{\psfig{figure=braid.eps,height=3.0cm,width=2.0cm}}
\caption{Tangles representing equation (\ref{eq:b3}).}
\label{fig:tangles}
\end{figure}
```

1.5 Bibliography

Please use the following style for the bibliography:

References

[1] R. Abłamowicz, Clifford algebra computations with Maple, *Clifford (Geometric) Algebras,* Banff, Alberta Canada, 1995, Ed. W. E. Baylis, Birkhäuser, Boston, 1996, 463–501.

[2] R. Abłamowicz and B. Fauser, Hecke algebra representations in ideals generated by q-Young Clifford idempotents, in *Clifford Algebras and their Applications in Mathematical Physics*, Eds. Rafał Abłamowicz and Bertfried Fauser, Vol. 1: Algebra and Physics, Birkhäuser, Boston, 2000, pages, 245–268.

[3] R. Abłamowicz, P. Lounesto, On Clifford algebras of a bilinear form with an antisymmetric part, in *Clifford Algebras with Numeric and Symbolic Computations,* Eds. R. Abłamowicz, P. Lounesto, J. M. Parra, Birkhäuser, Boston, 1996, 167–188.

1.6 Appendices

Before the appendices, please insert these commands:

```
\renewcommand{\thesection}{\Alph{section}}
\setcounter{section}{0}
\renewcommand{\theequation}{\Alph{section}.\arabic{equation}}
\section{Appendix}
```

that give new numbering for the equations in each appendix as shown next:

A Appendix

This is an equation in the appendix

$$f(x) = \cos(x) \tag{A.1}$$

A.1 Bibliography

Please follow the bibliography style and spacing between the bibliography items shown below. In order to set the spacing and font size, please copy and paste the following definitions right after `\begin{thebibliography}{99}` command:

```
\def\topsep{0pt}
\def\parsep{0pt plus 5pt minus 1pt}
\def\itemsep{-0.5ex} %a nice small skip between items
\small               %seems to be the best
```

References

[1] R. Abłamowicz, Clifford algebra computations with Maple, *Clifford (Geometric) Algebras,* Banff, Alberta Canada, 1995, Ed. W. E. Baylis, Birkhäuser, Boston, 1996, 463–501.

[2] R. Abłamowicz and B. Fauser, Hecke algebra representations in ideals generated by q-Young Clifford idempotents, in *Clifford Algebras and their Applications in Mathematical Physics*, Eds. Rafał Abłamowicz and Bertfried Fauser, Vol. 1: Algebra and Physics, Birkhäuser, Boston, 2000, pages, 245–268.

[3] R. Abłamowicz and B. Fauser, Eds., *Clifford Algebras and their Applications in Mathematical Physics*, Vol. 1: Algebra and Physics, Birkhäuser, Boston, 2000.

[4] R. Abłamowicz and P. Lounesto, On Clifford algebras of a bilinear form with an antisymmetric part, in *Clifford Algebras with Numeric and Symbolic Computations,* Eds. R. Abłamowicz, P. Lounesto, J. M. Parra, Birkhäuser, Boston, 1996, 167–188.

[5] R. Abłamowicz, 'CLIFFORD' - Maple V package for Clifford algebra computations, ver. 4, `http://math.tntech.edu/rafal/cliff4/`.

[6] E. Artin, Theory of braids, *Ann. Math.* **48** (1947), 101–126.

[7] C. Chevalley, *The Algebraic Theory of Spinors,* Columbia University Press, New York, 1954.

[8] A. Connes, *Noncommutative Geometry,* Academic Press, San Diego, 1994, Ch. V. 10.

[9] P. Lounesto; *Clifford Algebras and Spinors*, Cambridge University Press, Cambridge, 1997.

[10] J. Ryan and W. Sprößig, Eds., *Clifford Algebras and their Applications in Mathematical Physics*, Vol. 2: Clifford Analysis, Birkhäuser, Boston, 2000.

A.2 Information about the Authors

Please insert information about the Author(s) and the date of submission at the end of the paper using `\small` print:

```
%%%%%%%%%%%%%%%%%%%%%%information about author(s) here
\small
\vskip 1pc
{\obeylines
\noindent Rafa\l \ Ab\l amowicz
\noindent Department of Mathematics, Box 5054
\noindent Tennessee Technological University
\noindent Cookeville, Tennessee, 38505
\noindent E-mail: rablamowicz@tntech.edu
\vskip 1pc
\noindent Jane Liu
\noindent Department of Civil and Environmental Engineering, Box 5015
\noindent Tennessee Technological University
\noindent Cookeville, Tennessee, 38505
\noindent E-mail: jliu@tntech.edu
}
\vskip 6pt
\noindent Submitted: \today; Revised: TBA.

%%Adding one extra blank page if paper ends on odd page. Needed by the Editors.
%\clearpage
```

```
%\pagestyle{empty}
%\hspace{2.in}
```

which gives

Rafał Abłamowicz
Department of Mathematics, Box 5054
Tennessee Technological University
Cookeville, Tennessee, 38501
E-mail: rablamowicz@tntech.edu

Jane Liu
Department of Civil and Environmental Engineering, Box 5015
Tennessee Technological University
Cookeville, Tennessee, 38505
E-mail: jliu@tntech.edu

Submitted: December 7, 2005; Revised: TBA.

Finally, please enter command `\printindex` right before the `\end{document}` to produce index file for the paper:

```
\printindex
\end{document}
```