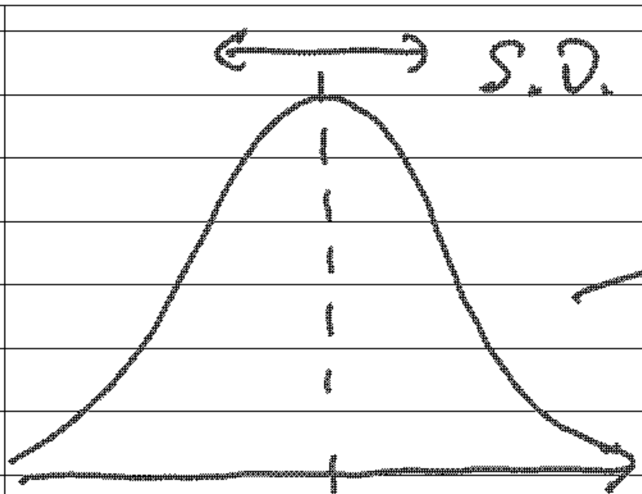
t-Distribution

Note Title 10/1/2008

S.D. σ is unknown



μ is unknown

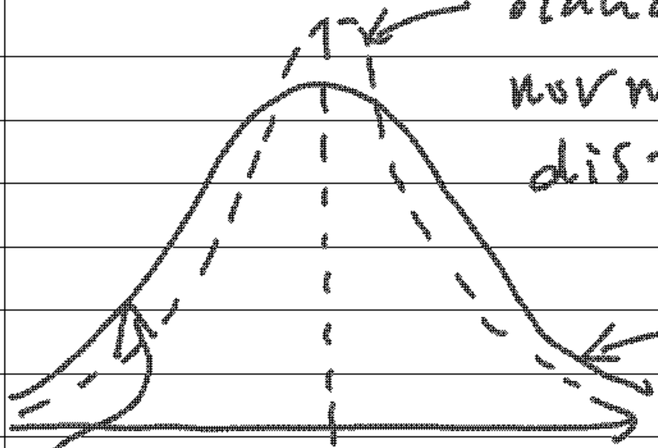
$X_1, \dots, X_n$

$\Downarrow$

$$\bar{X} = \frac{X_1 + \cdots + X_n}{n}$$

$\Downarrow$

standard normal distribution



0

t- distribution

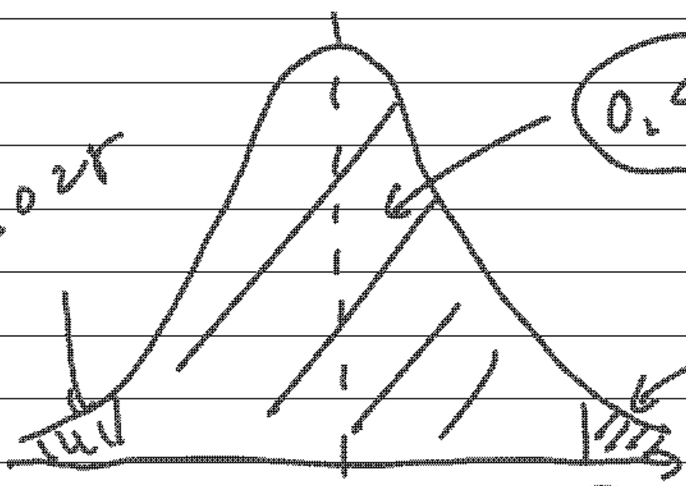
$$T = \frac{\bar{X} - \mu}{S/\sqrt{n}}$$

The shape is dependent of degree of freedom $\underset{(n-1)}{\shortparallel}$

$0.95 = 1 - \alpha \Rightarrow \alpha = 0.05$

0.025

$0.025 = \alpha/2$



$-t$ 0 t = critical point with level $0.025 = \alpha/2$

Confidence interval with unknown S.D.

$$P\left(-t_{\alpha/2} < T = \frac{\bar{X}-\mu}{S/\sqrt{n}} < t_{\alpha/2}\right) = \underset{\underset{0.95}{\shortparallel}}{(1-\alpha)}$$

The statement

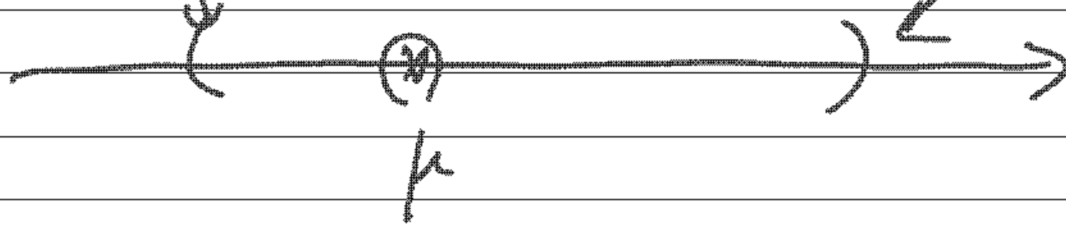
$$-t_{\alpha/2} < \frac{\bar{X}-\mu}{S/\sqrt{n}} < t_{\alpha/2}$$

is true $\underset{\underset{95\%}{\shortparallel}}{(1-\alpha)\%}$ of the time

Equivalently the statement

$$\underline{\bar{X} - t_{\alpha/2}\frac{S}{\sqrt{n}}} < \mu < \underline{\bar{X} + t_{\alpha/2}\frac{S}{\sqrt{n}}}$$

is true $(1-\alpha)\%$ of the time.



2. The gross profit margin of small businesses is estimated in a city. A random sample of 10 small businesses shows that the mean gross profit margin and their standard deviation are 5.2% and 7.5%, respectively.

(a) What are the limitations in using the confidence interval when the sample size is small?

You cannot assume that $\sigma = S$

Instead you have to use S in the C.I. when the sample size n is small.

In practice, if $n < 30$ then n is considered small.

(b) State the necessary assumptions, and construct 95% confidence interval for the mean gross profit margin of all small businesses in the city.

The population distribution must be normal. (But you don't assume σ)