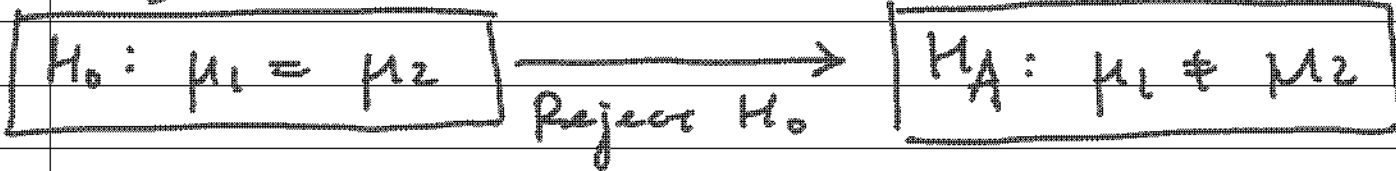


Two-sided test

Claim: $\mu_1 \neq \mu_2$



μ_1 = the average oxygen reading above town

μ_2 = " below town

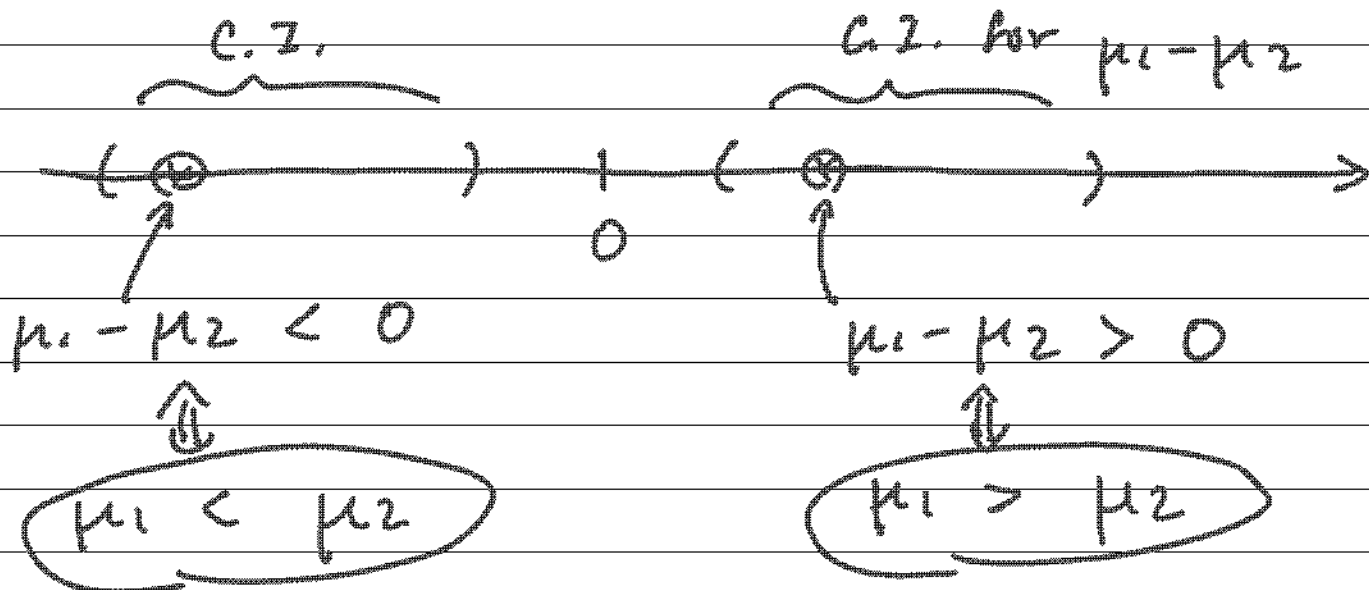
1. Case that H_0 is not rejected. No evidence to suggest $\mu_1 \neq \mu_2$.

2. Case that H_0 is rejected. There is evidence to suggest $\mu_1 \neq \mu_2$. It means either

$\mu_1 > \mu_2$ or $\mu_1 < \mu_2$.

↓
How can we determine?

↓
Use the C.I. for the difference $\mu_1 - \mu_2$



2. Answer the following questions regarding the study in Exercise 6.5. (Data file: Chapter 6/ex6-5.csv)

(a) Present a short description of the study and the data, including summary statistics for each variable.

The release of semitreated sewage into a river changes the level of dissolved oxygen of the river. 15 randomly selected specimens of river water were drawn at a location above the town, and another 15 specimens below the town. The dissolved oxygen readings (ppm, parts per million) are given in the table below.

Variables:

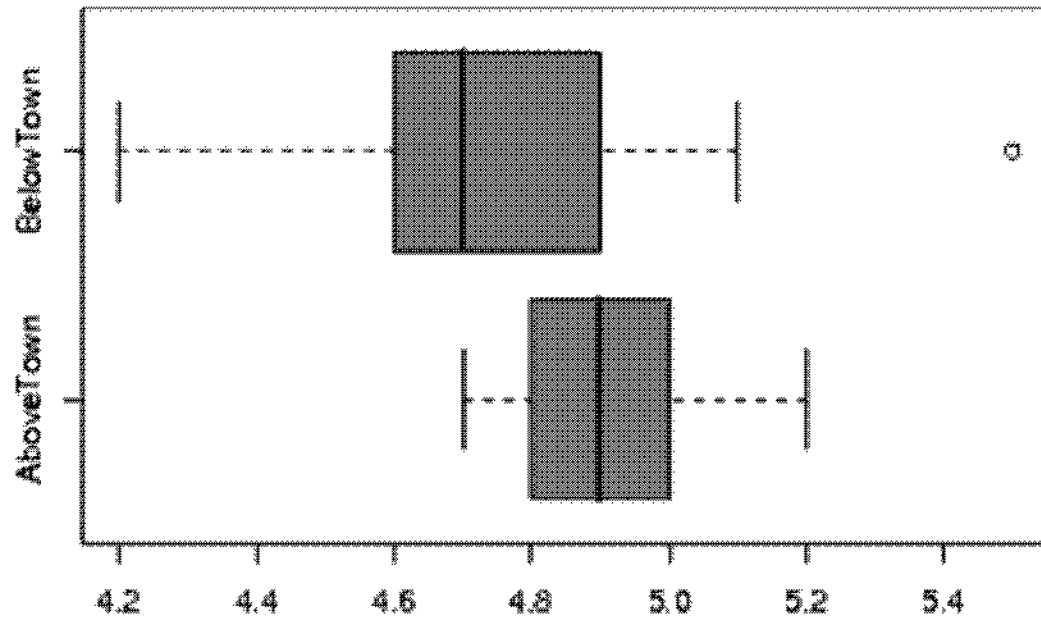
AboveTown: The dissolved oxygen readings above the town.

BelowTown: The dissolved oxygen readings below the town.

Summary statistics:

Variable	Mean	S.D	L.Quartile	Median	U.Quartile
AboveTown	4.92	0.156753	4.8	4.9	5
BelowTown	4.74	0.320268	4.6	4.7	4.9

(b) Present the comparison of the two groups in boxplot. Then comment on the data based on the visualization.



Data in "below town" have an outlier, and may not be symmetric. Data from "above town" may be assumed from normal distribution. Data from "above town" are shifted toward the right, indicating higher oxygen readings compared to "below town" data.

(c) Do the data provide sufficient evidence to indicate a difference in mean oxygen content between locations above and below the town? Construct the null and the alternative hypothesis for the test.

$$H_0: \mu_1 = \mu_2$$

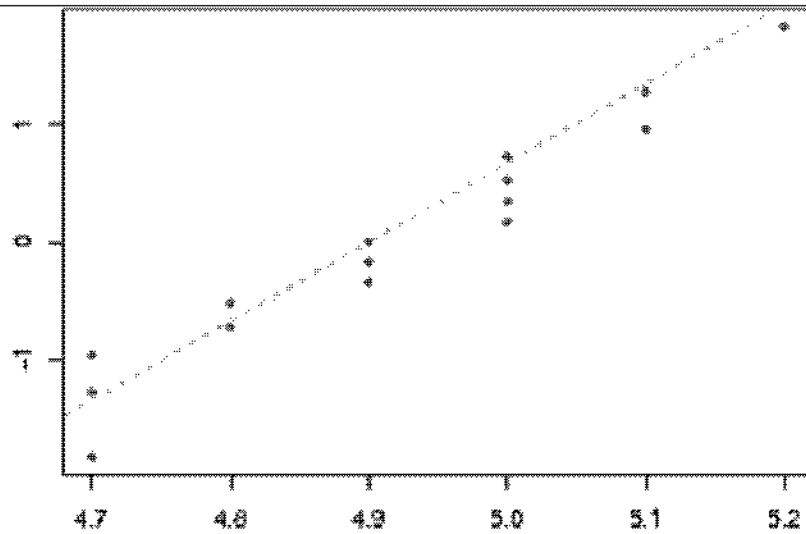
$$H_A: \mu_1 \neq \mu_2$$

μ_1 = the average oxygen reading above town

μ_2 = " " below town

(d) Construct QQ normal plot for each group. Do the normality assumption for the t-test appear to be valid for this study?

QQ plot for the oxygen readings at the location above the town

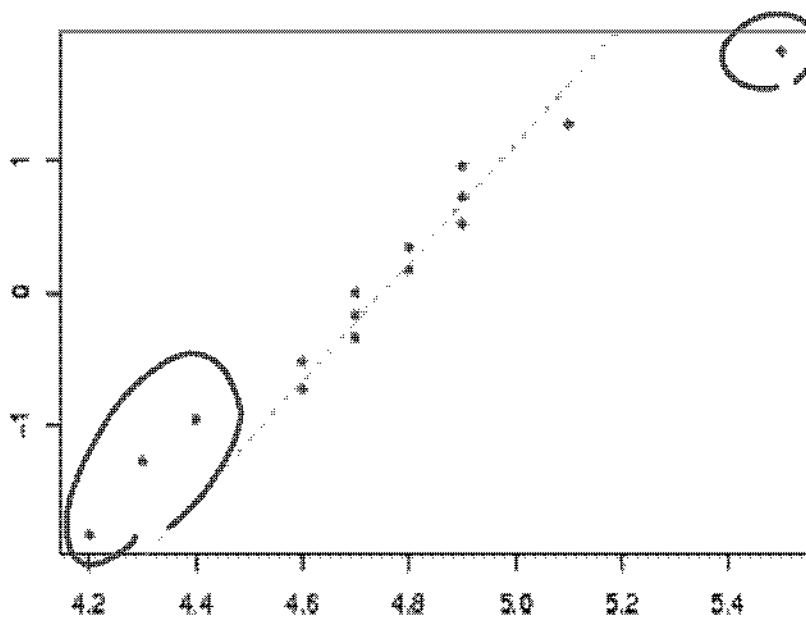


slope is different
from the indicated line



Due to the skewness
of sample data

QQ plot for the oxygen readings at the location below the town



Plots are off the line

There are some deviations from a normal distribution in both of the group.

(e) Calculate the test statistic and the p-value of the test.

Test procedure	t. statistic	p. value
General	1.95511	0.06444882
Pooled	1.95511	0.0606188

(f) Choose the significance level, and interpret your findings.

1. If $\alpha = 0.05$, we cannot reject H_0 .

⇒ No evidence to support the difference in oxygen reading.

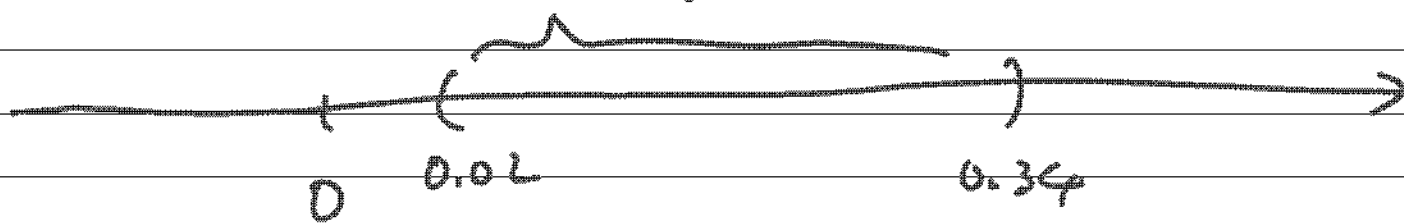
2. If $\alpha = 0.1$, we can reject H_0 .

⇒ There is some evidence to support the difference

The result indicates some evidence, but not significant.

(g) How large is the difference between the mean oxygen content above and below the town?

90% C.I. for $\mu_1 - \mu_2$ is (0.02, 0.34)



$$\mu_1 - \mu_2 > 0$$

⇓

$$\mu_1 (\text{above town}) > \mu_2 (\text{below town})$$

1. If $\alpha = 0.05$, 95% confidence interval suggests no significance for the difference in oxygen reading.

2. If $\alpha = 0.1$, 90% C.I. suggests that the oxygen reading is higher at the locations above the town.

Let $\alpha = 0.05$

Two-sided test

vs. One-sided test

$$H_A: \mu_1 \neq \mu_2$$

↓

$$\text{p-value} = 0.06 > \alpha$$

$$H_A: \mu_1 > \mu_2$$

↓

$$\text{p-value} = 0.03 < \alpha$$

↓
(1- α)% C.I. for $\mu_1 - \mu_2$

↓
Evidence that $\mu_1 > \mu_2$

↓
When H_0 is rejected, determine whether

$$\mu_1 > \mu_2 \text{ or } \mu_1 < \mu_2$$

But no evidence that $\mu_1 \neq \mu_2$