Correction. Page 12: rename "Theorem Conjecture 4" to Proposition 31.

Clarification. In Definition 3.5: "$x \in X$ for some $X \in \mathcal{C}$" means that "there exists some $X \in \mathcal{C}$ such that $x \in X$".

§ 4 *CONNECTEDNESS.*

4.1 Definition. Let A be a subset of a metric space X. Then A is *disconnected* means that $A = B \cup C$, where B and C are non-empty and neither B nor C contains a point or limit point of the other. In this case we say that the pair $\{B, C\}$ forms a *separation* of A. The set A is *connected* means that it is not disconnected. The space X is a *connected metric space* means that X is a connected subset of itself.

Theorem 32. *$\mathbb{R}$ is a connected metric space.*

Theorem 33. *A subset of $\mathbb{R}$ is connected iff it is an interval, i.e.: (a, b), $[a, b]$, $(a, b]$, $[a, b)$, $[a, \infty)$, (a, ∞), $(-\infty, a]$, $(-\infty, a)$, or $(-\infty, \infty)$.*

Theorem 34. *$\mathbb{R}^2$ is a connected metric space.*

4.2 Definition. A subset of a metric space is *clopen* means that it is simultaneously open and closed.

Note that $\varnothing$ is clopen, and that in the metric space X, X itself is a clopen set.

Theorem 35. *A metric space X is connected iff $\varnothing$ and X are the only clopen sets in X.*

Note that a pair $\{B, C\}$ of non-empty sets is a separation of $B \cup C$ iff $B \cap \mathrm{cl}(C) = \mathrm{cl}(B) \cap C = \varnothing$. On the other hand $\{(0,1), (1,2)\}$ is a separation of its union, so we can't simplify this condition to $\mathrm{cl}(B) \cap \mathrm{cl}(C) = \varnothing$. This example also shows that the union of two connected sets may be disconnected.

Proposition 36. *If A and B are connected sets and $A \cap B \neq \varnothing$ then $A \cup B$ is connected and $A \cap B$ is connected.*

Proposition 37. *If $A_1, A_2, A_3, \ldots$ is an infinite sequence of connected sets such that $A_1 \supset A_2 \supset A_3 \supset \ldots$, then $\bigcap_{i=1}^{\infty} A_i$ is connected.*

Proposition 38. *If A is connected, then $\mathrm{cl}(A)$ is connected.*