
1) (10 pts) Rework problem 3 from Quiz 6 *without* using polar coordinates. You should probably use a computer algebra system to do this, although it should be possible to carry out the calculations by hand if you are careful and persevering (feel free to use an integral table in this case).

- a) Let m_1 and m_2 denote the slopes of the lines with angles $1/2$ radian and 1 radian respectively. Let a be the x -coordinate of the point where the line of slope m_2 intersects the circle of radius 2 . Let b be the x -coordinate of the point where the other line intersects the circle. Use a bit of trigonometry to express the values of m_1 , m_2 , a , and b . Also compute approximate values for these constants in order to check that your answers are reasonable.
- b) Split the “polar rectangular” region D into two regions: let D_1 be the portion of D to the left of the line $x = a$, and let D_2 be the portion to the right. Now write each of the integrals $\iint_{D_1} x^2 + y^2 dA$ and $\iint_{D_2} x^2 + y^2 dA$ as an iterated integral using “ $dy dx$ ” order.
- c) Evaluate each of the integrals in (b). This is where you probably want to use Maple or a similar computer algebra system. Use the exact (not approximate) values for m_1 , m_2 , a , and b . Your answers should have lots of terms involving $\tan(1/2)$, $\cos(1)$, etc. Then compute approximate values for these two integrals as well.
- d) Add together the results of the exact computations in (c) and then simplify to show that the result does agree with the calculation $\iint_D x^2 + y^2 dA = 2$ from quiz 6. Maple’s “simplify” command is probably the best way to do this.
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Extra Credit: It is easy to generalize this problem a bit. Let θ_1 and θ_2 be any angles satisfying $0 < \theta_1 < \theta_2 < \pi/2$, and let D denote the region in the first quadrant between the lines making angles θ_1 and θ_2 with the x -axis and inside the circle of radius R , where $R > 0$. Using polar coordinates, it is easy to compute that:

$$\iint_D x^2 + y^2 dA = \int_{\theta_1}^{\theta_2} \int_0^R r^3 dr d\theta = \frac{R^4}{4}(\theta_2 - \theta_1)$$

Derive this formula without using polar (or cylindrical) coordinates.