- *Vectors in* $\mathbb{R}^2$ *and* $\mathbb{R}^3$

1) Compute $2\langle 1,3\rangle - 4\langle 2,1\rangle$, and sketch a picture showing the relative positions of the relevant vectors.

2) Compute the area of the triangle in $\mathbb{R}^3$ with vertices at $(1,1,1)$, $(2,-1,3)$, and $(0,1,1)$. Find an equation for the plane which contains this triangle. Compute the angle between the normal vector to this plane and the vector $\langle 1,1,1\rangle$.

- *Functions* $f : \mathbb{R} \to \mathbb{R}^n$

3) Find a parameterization $\vec{r}$ for the line L in $\mathbb{R}^3$ which contains the points $(1,2,-1)$ and $(2,2,3)$. Find a set of symmetric equations for L also. Does L intersect the plane from problem #2? If so, where?

4) Sketch the curve $\vec{r}(t) = \left\langle 2^t, t^2 \right\rangle$. Find the equation of the line tangent to the curve at the point $\vec{r}(3)$. Find an integral for the length of the section of this curve which is between the points $(1,0)$ and $(16,16)$. (Hint: it may help to recall that $2^t \overset{\text{def}}{=} e^{\ln(2)t}$.)

- *Functions* $f : \mathbb{R}^n \to \mathbb{R}$

5) Compute $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^3 - y^2}{x^2 + y^2}$ and $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^3 - y^2 + xy^2 - x^2}{x^2 + y^2}$.

6) Let $f(x,y) = 2\sin(x)\cos(y) + 6\cos(x)\sin(y)$. Find the equation for the plane tangent to the surface $z = f(x,y)$ at the point on the graph whose x and y coordinates are both equal to $\pi/4$. Compute the volume which lies below this surface and above the rectangle in the x-$y$plane with vertices at $(0,0,0)$, $(0,\pi/4,0)$, $(\pi/4,0,0)$, and $(\pi/4,\pi/4,0)$. Find a unit vector in $\mathbb{R}^2$ in the direction of maximal increase for f at the point $(\pi/4,\pi/4)$.

7) Let $f(x,y,z) = xe^{yz}$. Compute the directional derivative of f at the point $(1,2,0)$ in the direction of the vector $\langle -1,1,1\rangle$. Find an equation for the plane tangent to the surface $xe^{yz} = e^6$ at the point $(1,2,3)$. Set up an integral for $\iiint_E f\,dV$, where $E = \{\,(x,y,z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 \le 2 \text{ and } x \ge 0, y \ge 0, z \ge 0\,\}$.

(over)

8) Explain the relationships between the differentiability of f, the continuity of f, and the continuity of the partial derivatives of f.

9) Use differentials to approximate $\dfrac{3.012\sqrt{100.005}}{9.998}$

10) Use the chain rule to find $f'(2)$ given that $f(t) = g(x(t), y(t))$ and that $x(2) = 3$, $x'(2) = 4$, $y(2) = 5$, $y'(2) = 6$, $g_x(3,5) = 7$, $g_x(4,6) = 8$, $g_y(3,5) = 9$, and $g_y(4,6) = 10$. (Where g_x and g_y are the partial derivatives of g with respect to its first and second variable, respectively.)

11) Find the critical points of $f(x,y) = (x^4 + y^2)e^y$ and apply the second derivative test at each one. Does f have any local extrema?

12) Compute $\iint\limits_D x^2y^2\,dA$, where D is the region sketched below.

The printed sheet has a hand-drawn diagram showing D as the region above $y = 1$ and below $y = \sqrt{x}$ with x between 1 and 4.

- *Functions* $f : \mathbb{R}^n \to \mathbb{R}^n$

13) Let $\vec{f} = \langle x\sin(y),\, y\cos(x)\rangle$. Compute the Jacobian matrix for $\vec{f}$, and find its determinant.

14) Show that the vector field $\vec{f}(x,y) = \langle \sin(x)\sin(y),\, \cos(x)\cos(y)\rangle$ is *not* conservative.

15) Show that the vector field $\vec{f}(x,y) = \langle e^x + y,\, e^y + x\rangle$ is conservative, and find a scalar function g such that $\vec{f} = \vec{\nabla} g$. Let C denote the section of curve in the last part of problem #4. Set up, but do not evaluate, a definite integral of one variable which is equal to $\int_C \vec{F} \cdot d\vec{r}$. Your answer should look like a Calc-II problem and should not have any vector notation. Briefly explain how each of the terms of this integral could be anti-differentiated by a technique from Calc-II. Compute the value of $\int_C \vec{F} \cdot d\vec{r}$ by using the fundamental theorem for line integrals instead of the fundamental theorem of calculus.

FINAL EXAM: Wednesday, Dec 13, 3:30–5:30pm

REVIEW SESSION: Tuesday, Dec 12, 6pm