## AN ABSTRACT OF A THESIS

## TYPING THESES AND DISSERTATION AT TTU

**Rafal Ablamowicz**

**Master of Science in Mathematics**

Singularity avoidance is of great importance for the proper control of a robotic arm. In a hyper-redundant robot the singularities are much harder to detect and the Clifford algebra can be used to parameterize a singularity space.

# TYPING THESES AND DISSERTATION AT TTU

A Thesis

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In Partial Fulfillment

of the Requirements for the Degree

MASTER OF SCIENCE

Mathematics

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**CERTIFICATE OF APPROVAL OF THESIS**

**TYPING THESES AND DISSERTATION AT TTU**

**by**

**Rafal Ablamowicz**

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## DEDICATION

This thesis is dedicated to my wife or husband or parents whose encouragement have meant to me so much during the pursuit of my graduate degree and the composition of the thesis. This thesis is dedicated to my wife or husband or parents whose encouragement have meant to me so much during the pursuit of my graduate degree and the composition of the thesis. This thesis is dedicated to my wife or husband or parents whose encouragement have meant to me so much during the pursuit of my graduate degree and the composition of the thesis. This thesis is dedicated to my wife or husband or parents whose encouragement have meant to me so much during the pursuit of my graduate degree and the composition of the thesis.

## ACKNOWLEDGMENTS

I would like to thank all Dr. Ralph Nimoy, Dr. Mike Spock, and Professor Y2K for their most valuable help in writing this thesis. Hope they will help me write another thesis later...

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# CHAPTER 1
# INTRODUCTION

## 1.1 Revisions to `ttuthesis.sty`

In this new `ttuthesis.sty` file dated 12-3-2010, on the Copyright page TTU Graduate School now requires Roman number ii to appear.

Previous revision was done on 12-03-2007 when two bugs were fixed in the definition of `\section` and `\subsection` commands.

- The first bug had to do with the older definitions not allowing for a proper labeling of sections as in `\label{sectionlabel}` and subsections as in `\label{subsectionlabel}`.

  That is, when using `\ref{sectionlabel}` and `\label{subsectionlabel}` in the text, only the current chapter number was printed and not the section or the subsection number.

For example, right after the title of the current section, I have inserted `\label{revisions}` so that I could refer to this section later in the text as in `\ref{revisions}` which produces a correct section number 1.1. Likewise, for subsections: The first subsection below has been labeled as `\label{Young}` and it can now be referred as subsection 1.2.1.

Remember that per TTU Graduate School requirements, the subsubsections remain unnumbered: hence, they cannot be refered to with `\ref` command even if they are labeled with `\label` command.

- The second bug had to do with not allowing for a use of an optional parameter with a shorter title as in `\section`, `\subsection`, and `\subsubsection` commands.

If you look at the TeX code of `chapter1.tex` you will find that the title of the current section has been typed in as

```
\section[Revisions]{Revisions to {\tt ttuthesis.sty}}
```

with the REQUIRED now parameter showing a shorter title `\section[Revisions]` showing up between the brackets first and the longer title showing up between the braces.

The shorter title appears now in the Table of Contents whereas the longer title appears now in the actual text of the thesis. If you want to have both the same, that's fine, just use for the shorter title the longer title.

This new parameter containing now the short title MUST also be used in titles of subsections and subsubsections: If not used, the following error will result when typesetting:

```
./Thesis.lof) [10] (./chapter1.tex
CHAPTER 1.
! Use of \section doesn't match its definition.
l.6 \section{
             Revisions to {\tt ttuthesis.sty}}
?
```

I have caused that error to appear by removing that required now parameter `[Revisions]` from the command `\section` in

```
\section[Revisions]{Revisions to {\tt ttuthesis.sty}}
```

Thus, to summarize, to avoid typesetting errors, you MUST now insert these optional parameters with short titles in all commands `\section`, `\subsection`, and `\subsubsection`. If you want the same title to appear in the Table of Contents, copy the longer title as the shorter title as well. For example, the title of the next section 1.2 is typed up as:

```
\section[Hyper-redundant robots]{Hyper-redundant robots}
\label{robots}
```

In the command `\chapter`, the use of this short title is optional. This is because the command `\chapter` was defined earlier by authors of this style file with greater care and fixing these definitions would require a major rewrite of the file.

If you have any questions, please contact me via email at `rablamowicztntech.edu`.
Cookeville, December 3, 2011

## 1.2 Hyper-redundant robots

Hyper-redundant robots have many degrees of freedom (DOF) and are sometimes called snake or worm robots. Clifford algebra is a specially described algebra which can resolve rotations and translations without matrix algebra. For a definition of the Clifford algebra see [1] and references therein. Consider the Clifford algebra $C\ell_{3,0}$, also denoted as $C\ell_3$, over $\mathbb{R}^3$ endowed with a quadratic form $q = \mathrm{diag}(1,1,1)$. Let $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ be an orthonormal basis in $\mathbb{R}^3$. Then, there is the following famous relation in the Clifford algebra $C\ell_3$ :

$$\mathbf{e}_i\mathbf{e}_j + \mathbf{e}_j\mathbf{e}_i = 2\delta_{i,j} \tag{1.1}$$

where $\delta_{ij}, i,j = 1,2,3$ is the Kronecker delta function. Notice that relation (1.1) reduces to

$$\mathbf{e}_i^2 = 1, \quad i = 1,2,3. \tag{1.2}$$

The aim of this work is to provide a mechanism for breaking up the ordinary spinor representation of $C\ell_{n,n}$ into tensor products of smaller representations using appropriate Young operators constructed as Clifford idempotents. This means that a suitable Clifford algebra is used as a carrier space for various tensor product representations.

### 1.2.1 Young Operators Long

The Young operators for various Young diagrams provide a set of idempotents which decompose the unity $\mathbf{1}$ of $C\ell_{n,n}$ as

$$\mathbf{1} = Y^{(\lambda_1)} + \cdots + Y^{(\lambda_n)}, \tag{1.3}$$

where $(\lambda_i)$ is a partition of $n$ characterizing the appropriate Young tableau, that is, a Young diagram (frame) with an allowed numbering. Let $Y^{(\lambda_i)}_{i_1,\ldots,i_n}$ denote a Young tableaux where $(\lambda_i)$ is an ordered partition of $n$ and $i_1,\ldots,i_n$ is an allowed numbering of the boxes in the Young diagram corresponding to $(\lambda_i)$ as in [23, 30]. Furthermore, these Young operators are mutually annihilating idempotents

$$Y^{(\lambda_i)}Y^{(\lambda_j)} = \delta_{\lambda_i\lambda_j}Y^{(\lambda_j)}. \tag{1.4}$$

It appears natural to ask if these Young operators can be used to give representations of the symmetric group *within* the Clifford algebraic framework. The representation spaces which appear as a natural outcome of the embedding of the symmetric group, and its representations can then be looked at as multi-particle spinor states. However, these might not be spinors of the full Clifford algebra.

In order to be as general as possible, in the following, not only the representations of the symmetric group will be considered, but also of the Hecke algebra $H_{\mathbb{F}}(n,q)$. The Hecke algebra is the generalization of the group algebra of the symmetric group by adding the requirement that transpositions $t_i$ of *adjacent* elements $i, i+1$ are no longer involutions $s_i$. Equation $t_i^2 = (1-q)t_i + q$ reduces to $s_i^2 = \mathbf{1}$ in the limit $q \to 1$.

Hecke algebras are 'truncated' braids since a further relation (see (1.5) below) is added to the braid group relations as in [4]. A detailed treatment of this topic with important links to physics may be found, for example, in [21, **?**] and in the references of [12].

The defining relations of the Hecke algebra will be given according to Bourbaki [9]. Let $< \mathbf{1}, t_1, \ldots, t_n >$ be a set of generators which fulfill these relations:

$$t_i^2 = (1-q)t_i + q, \tag{1.5}$$

$$t_i t_j = t_j t_i, \quad |i-j| \geq 2, \tag{1.6}$$

$$t_i t_{i+1} t_i = t_{i+1} t_i t_{i+1}. \tag{1.7}$$

Then their algebraic span is the Hecke algebra. Since the results in this thesis will be compared with those of King and Wybourne [29] – hereafter denoted by KW – one needs to provide a transformation to their generators $g_i$, namely $g_i = -t_i$, which results in a new quadratic relation

$$g_i^2 = (q-1)g_i + q \tag{1.8}$$

while the other two remain unchanged. However, this small change in sign is responsible for great differences especially in the $q$-polynomials occurring in both formulas. One immediate consequence is that this transformation interchanges symmetrizers and antisymmetrizers. In particular, this replacement connects Formula (3.4) in KW with full symmetrizers while Formula (3.3) in KW gives full antisymmetrizers. Finally, the algebra morphism $\rho$ which maps the Hecke algebra into the even part of an appropriate Clifford algebra can be found in [12].

Let $\{\mathbf{1}, \mathbf{e}_1, \ldots, \mathbf{e}_{2n}\}$ be a set of generators of the Clifford algebra $C\ell(B,V)$ where the vector space $V = \{\mathbf{e}_i\} =< \mathbf{e}_i >$ is endowed with a non-symmetric $2n \times 2n$ bilinear form $B = [B(\mathbf{e}_i, \mathbf{e}_j)] = [B_{i,j}]$ defined as

$$B_{i,j} := \begin{cases} 0, & \text{if } 1 \leq i,j \leq n \text{ or } n < i,j \leq 2n, \\ q, & \text{if } i = j-n \text{ or } i-1-n = j, \\ -(1+q), & \text{if } i+1 = j-n \text{ or } i = j+1-n, \\ -1, & \text{if } |i-j-n| \geq 2 \text{ and } i > n, \\ 1, & \text{otherwise.} \end{cases} \tag{1.9}$$

The most general case would have $\nu_{ij} \neq 0$ in the last line of (1.9). For example, when $n = 4$, then

$$B = \begin{pmatrix} 0 & 0 & 0 & 0 & q & -1-q & 1 & 1 \\ 0 & 0 & 0 & 0 & -1-q & q & -1-q & 1 \\ 0 & 0 & 0 & 0 & 1 & -1-q & q & -1-q \\ 0 & 0 & 0 & 0 & 1 & 1 & -1-q & q \\ 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 \\ q & 1 & 1 & -1 & 0 & 0 & 0 & 0 \\ -1 & q & 1 & 1 & 0 & 0 & 0 & 0 \\ -1 & -1 & q & 1 & 0 & 0 & 0 & 0 \end{pmatrix}. \tag{1.10}$$

The bilinear form $B$ in (1.10) is our particular choice that guarantees that the following equations hold:

$$\rho(t_i) = b_i := \mathbf{e}_i \wedge \mathbf{e}_{i+n}, \tag{1.11}$$

$$b_i b_j = b_j b_i, \quad \text{whenever} \quad |i-j| \geq 2, \tag{1.12}$$

$$b_i b_{i+1} b_i = b_{i+1} b_i b_{i+1}. \tag{1.13}$$

This shows $\rho$ to be a homomorphism of algebras implementing the Hecke algebra structure in the Clifford algebra $C\ell(B,V)$. One knows from [12] that $\rho$ is not injective, and that its kernel contains all Young diagrams which are not L-shaped (that is, diagrams with at most one row and/or one column). The first instance, however, when this kernel is non-trivial, occurs in $S_4$ where the partition $4 = (2,2)$ gives a Young diagram of square form which is not L-shaped.

Testing section 2.3 here.

# CHAPTER 2
# THEORETICAL DEVELOPMENTS

This chapter explains the design technique developed in this research project. The first section bla bla bla. Ultimately the equation

$$E = m * c^2 \tag{2.1}$$

$$E = m * 9 * 10^{16} \tag{2.2}$$

In Section 2.1 we will show something. In Section 2.2 we will show something else, while in Section 2.3 we will discuss a new case.

In Subsection 2.1.1 we will show something. In Subsection 2.2.1 we will show something else, while in Subsection 2.3.1 we will discuss a new case.

## 2.1 Singularity Problems Longer

At a singularity, an infinite joint velocity is required to move the end effector in a particular direction as in Equation 2.2 [32]. Also near the singularity large forces can be found that can damage the robot or an object being worked on. Bla bla bla.

### 2.1.1 Singularity Issues Longer

#### Special Applications Longer.

## 2.2 Surface Parameterization Longer

Clifford algebras of the $C\ell^+$ can be used to bla bla bla.

### 2.2.1 Surface Subsection

## 2.3 The case of $H_{\mathbb{F}}(2, q)$ and $S_2$

Beginning with $H_{\mathbb{F}}(2, q)$ which reduces to $S_2$ in the limit $q \to 1$. $H_{\mathbb{F}}(2, q)$ is generated by $\{\mathbf{1}, b_1\}$. Thus, having only one $q$-transposition, from which a $q$-symmetrizer $R(12)$ and a $q$-antisymmetrizer $C(12)$ can be calculated.

### 2.3.1 Main Case

Notice that in the limit $q \to 1$ the following relations for a set of new generators defined as $s_i := \lim b_i$ when $q \to 1$ :

(i) $s_i^2 = 1$,

(ii) $s_i s_j = s_j s_i$, whenever $|i - j| \geq 2$,

(iii) $s_i s_{i+1} s_i = s_{i+1} s_i s_{i+1}$,

(iii)' $(s_i s_{i+1})^3 = \mathbf{1}$.

Property (iii)' follows from the fact that $s_i^2 = 1$ and $s_i^{-1} = s_i$. This is a presentation of the symmetric group according to Coxeter-Moser [7]. Now it is an easy matter to show that (ii) is valid for transpositions, and that (iii) can be calculated graphically using tangles as in Figure 1.

**Definition 1.** *An associative algebra over a field* $\mathbb{F}$ *with unity* $1$ *is the* Clifford algebra $C\ell(Q)$ *of a non-degenerate quadratic form* $Q$ *on a vector space* $V$ *over* $\mathbb{F}$ *if it contains* $V$ *and* $\mathbb{F} = \mathbb{F} \cdot 1$ *as distinct subspaces so that*

1. $\mathbf{x}^2 = Q(\mathbf{x})$ *for any* $\mathbf{x} \in V$,
2. $V$ *generates* $C\ell(Q)$ *as an algebra over* $\mathbb{F}$,
3. $C\ell(Q)$ *is not generated by any proper subspace of* $V$.

**Theorem 1.** *Suppose that* $1 \leq p < (1-\kappa)^{-1}$. *If* $u \in L^p(\mathbb{R}^n, \mathbb{R})$ *and* $u \geq 0$, *then*

$$|\mathcal{I}u(x)| \leq A_*(k,p)[\mathcal{M}_* u(x)]^{1-(1-\kappa)p} \|u\|_p^{(1-\kappa)p}, \qquad x \in \mathbb{R}^n. \tag{2.3}$$

*Proof.* Since $u \geq 0$, it is clear that

$$|k * u(x)| \leq \max\{|k_+ * u(x)|, |k_- * u(x)|\}. \tag{2.4}$$

By applying Theorem 1 to $k_+$ and $k_-$ one gets

$$|k_\pm * u(x)| \leq A(k_\pm, p)[\mathcal{M}_\pm u(x)]^{1-(1-\kappa)p} \|u\|_p. \tag{2.5}$$

It remains to combine (2.4) and (2.5) with (**??**) and (**??**).[1]

The reader can easily check that (2.3) is sharp. Moreover, if it is assumed that the kernel $k$ is an odd function, then $X_- = (-1)X_+$ and

$$\mathrm{vol}(X_+) = \mathrm{vol}(X_-) = \frac{1}{2}\mathrm{vol}(X);$$

hence,

$$A_*(k,p) = 2^{-\kappa} A(k,p). \tag{2.6}$$

[]

[1] These last two calls to references have resulted in (**??**) and (**??**) showing, and in an error message in your paper's log file that is automatically generated by PCTEX32. This is because references have been made to non-existing labels.

# CHAPTER 3

# INCLUDING FIGURES IN THESIS OR DISSERTATION

The math required to find the singularity space can be quite lengthy and errors are easily made. Therefore a computer program called CLIFFORD (see [3]) is used. In addition, it is useful to visualize the singularity space via a graph which can be generated by Maple. Thus, it is also useful to know how to include plots and figures in a LaTeX document.

The following will show how to include plots in your LaTeX document with a command `\includegraphics` which can handle many different file formats. For more information on basic LaTeX commands see [24] and for more advanced features of AMS-LaTeX see [22].

## 3.1 Developing a Set of Parameters

CLIFFORD can perform computations in any Clifford algebra; in particular, it can compute with quaternions. Quaternions give a convenient representation of rigid motions in a three dimensional real space $\mathbb{R}^3$.

## 3.2 Plotting Singularities

One can control vertical spacing before and after a figure with `\vskip` commands. For example, the following code produces Figure (3.1).

```
\begin{figure}[tb]
\begin{center}
\scalebox{0.8}{\includegraphics{plot03.eps}}
```

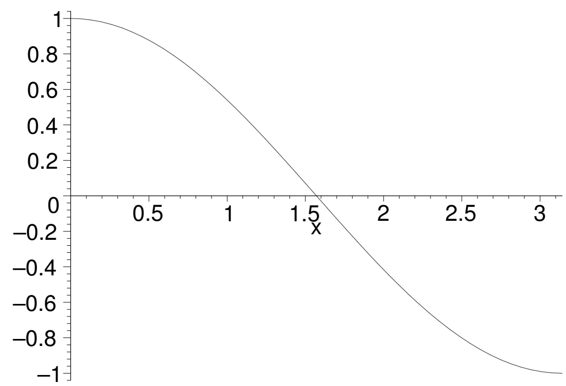


Figure 3.1. The wave of the future - first time

```
\end{center}
\caption{The wave of the future - first time}
\label{fig:sin}
\end{figure}
```

As it can be seen from the code that produces Figure 3.1, this is simple. Observe the extra scaling parameter in

```
\scalebox{0.8}{\includegraphics{plot03.eps}}
```

of 0.8. Changing it to a smaller value than 1.0, reduces the size of the figure whereas increasing to a value above 1.0, magnifies the size.

## 3.3 Using the Program to Avoid Singularities

It is so simple to use this program. As seen in Table 3.1. To display this table, the following code will be used.

```
\begin{table}[tb]
\caption{A sample table} \label{tab}
\vspace{0.2in}
\arrayrulewidth 1pt
\doublerulesep 5pt
\begin{center}
\begin{tabular}{ccc} \hline \hline
```

Table 3.1. A sample table

| Header 1 | Header 2 | Header 3 |
|---|---|---|
| 1.000 | 2.000 | 3.000 |
| 1.275 | 2.500 | 3.250 |
| 1.500 | 2.875 | 3.450 |

```
   Header 1 & Header 2 & Header 3 \\ \hline
   1.000 & 2.000 & 3.000 \\ \hline
   1.275 & 2.500 & 3.250 \\ \hline
   1.500 & 2.875 & 3.450 \\ \hline
\end{tabular}
\end{center}
\end{table}
```

Notice, that again one must make sure that the table will be placed at the top or at the bottom of a page. This is accomplished by using parameters `[tb]` as in `\begin{table}[tb]`. Then, the table will be placed centrally on a page, either at the top or at the bottom.

## REFERENCES

[1] R. Abłamowicz, P. Lounesto, On Clifford algebras of a bilinear form with an antisymmetric part, in *Clifford Algebras with Numeric and Symbolic Computations,* Eds. R. Abłamowicz, P. Lounesto, J. M. Parra, Birkhäuser, Boston, 1996, 167–188.

[2] R. Abłamowicz, Clifford algebra computations with Maple, *Clifford (Geometric) Algebras,* Banff, Alberta Canada, 1995, Ed. W. E. Baylis, Birkhäuser, Boston, 1996, 463–501.

[3] R. Abłamowicz, 'CLIFFORD' - Maple V package for Clifford algebra computations, ver. 4, `http://math.tntech.edu/rafal/cliff4/`.

[4] E. Artin, Theory of braids, *Ann. Math.* **48** (1947), 101–126.

[5] C. Chevalley, *The Algebraic Theory of Spinors,* Columbia University Press, New York, 1954.

[6] A. Connes, *Noncommutative Geometry,* Academic Press, San Diego, 1994, Ch. V. 10.

[7] H. S. M. Coxeter, W. O. J. Moser, *Generators and Relations for Discrete Groups, Fourth Edition,* Springer Verlag, Berlin, 1980.

[8] N. Bala, *Network Synthesis,* Englewood Cliffs, Prentice-Hall, 1958.

[9] N. Bourbaki, *Algebra 1,* Springer Verlag, Berlin, 1989, Chapters 1–3.

[10] R. Boudet, The Glashow-Salam-Weinberg electroweak theory in the real algebra of space time, in *The Theory of the Electron,* J. Keller, Z. Oziewicz, Eds., Cuautitlan, Mexico, 1995, *Adv. in Appl. Clifford Alg.* **7** (Suppl.) (1997), 321–336.

[11] C. Doran, A. Lasenby, S. Gull, States and operators in spacetime algebra, *Found. Phys.* **23** (9), (1993), 1239. S. Somaroo, A. Lasenby, C. Doran, Geometric algebra and the causal approach to multi-particle quantum mechanics, *J. Math. Phys.* Vol. **40**, No. **7** (July 1999), 3327–3340.

[12] B. Fauser, Hecke algebra representations within Clifford geometric algebras of multivectors, *J. Phys. A: Math. Gen.* **32** (1999), 1919–1936.

[13] B. Fauser, Clifford-algebraische formulierung und regularität der Quantenfeldtheorie, Thesis, Uni. Tübingen, 1996.

[14] B. Fauser, H. Stumpf, Positronium as an example of algebraic composite calculations, in *The Theory of the Electron,* J. Keller, Z. Oziewicz, Eds., Cuautitlan, Mexico, 1995, *Adv. Appl. Clifford Alg.* **7** (Suppl.) (1997), 399–418.

[15] B. Fauser, Clifford geometric parameterization of inequivalent vacua, Submitted to *J. Phys. A.* (hep-th/9710047).

[16] B. Fauser, On the relation of Clifford-Lipschitz groups to $q$-symmetric groups, *Proc. XXII Int. Colloquium on Group Theoretical Methods in Physics,* Hobart, Tasmania, 1998, Eds. S. P. Corney, R. Delbourgo and P. D. Jarvis, International Press, Cambridge, MA, 1999, 413–417.

[17] B. Fauser, Clifford algebraic remark on the Mandelbrot set of two-component number systems, *Adv. Appl. Clifford Alg.* **6** (1)(1996), 1–26.

[18] B. Fauser, On an easy transition from operator dynamics to generating functionals by Clifford algebras, *J. Math. Phys.* **39** (1998), 4928–4947.

[19] B. Fauser, Vertex normal ordering as a consequence of nonsymmetric bilinear forms in Clifford algebras, *J. Math. Phys.* **37** (1996), 72–83.

[20] W. Fulton, Joe Harris, *Representation Theory,* Springer, New York, 1991.

[21] D. M. Goldschmidt, *Group Characters, Symmetric Functions, and the Hecke Algebra,* University Lecture Series, Berkeley (Spring 1989) Providence, RI: AMS Vol. **4** (1993).

[22] G. Grätzer, *Math into LaTeX: An Introduction to LaTeXand AMS-LaTeX,* Birkhäuser, Boston, 1996.

[23] M. Hamermesh, *Group Theory and Its Application to Physical Problems,* Addison-Wesley, London, 1962.

[24] J. Hahn, *LaTeX for Everyone: A Reference Guide and Tutorial for Typesetting Documents Using a Computer,* Prentice Hall PTR, Upper Saddle River, New Jersey, 1993.

[25] D. Hestenes, *Spacetime Algebra,* Gordon and Beach, 1966.

[26] D. Hestenes, *New Foundation for Classical Mechanics,* Kluwer, Dordrecht, 1986.

[27] H. Hiller, *Geometry of Coxeter Groups,* Pitman Books Ltd., London, 1982.

[28] P. N. Hoffman, J. F. Humphreys, *Projective Representations of the Symmetric Groups,* Oxford University Press, Oxford, 1992.

[29] R. C. King, B. G. Wybourne, Representations and traces of Hecke algebras $H_n(q)$ of type $\mathbf{A}_{n-1}$, *J. Math. Phys.* **33** (1992), 4–14.

[30] I. G. Macdonald, *Symmetric Functions and Hall Polynomials,* Oxford University Press, Oxford, 1979.

[31] S. Majid, *Foundations of Quantum Group Theory,* Cambridge University Press, Cambridge, 1995.

[32] J.M. McCarthy, *An Introduction to Theoretical Kinematics,* Massachusetts Institute of Technology, MIT Press, 1990.

[33] Z. Oziewicz, From Grassmann to Clifford, in *Proceedings Clifford Algebras and Their Application in Mathematical Physics,* Canterbury, UK, J. S. R. Chisholm, A. K. Common, Kluwer, Dordrecht, 1986, 245–256.

[34] Z. Oziewicz, *Clifford Algebra of Multivectors,* in *Theory of the Electron,* Eds. J. Keller and Z. Oziewicz, *Adv. in Appl. Clifford Alg.* **7** (Suppl.) (1997), 467–486.

[35] Z. Oziewicz, Clifford algebra for Hecke braid, in *Clifford Algebras and Spinor Structures,* Special Volume to the Memory of Albert Crumeyrolle, Eds. R. Abłamowicz, P. Lounesto, Kluwer, Dordrecht, 1995, 397–412.

[36] R. Penrose, W. Rindler, Spinors and space-time: two-spinor calculus and relativistic fields, *Cambridge Monographs on Mathematics,* Cambridge University Press, Paperback Reprint Edition, Vol. **1**, 1987.

[37] J. Rotman, *Galois Theory,* Springer, New York, 1990.

**APPENDICES**

# APPENDIX A: THIS IS MIKE'S FIRST APPENDIX

In addition to the main package CLIFFORD, in Sections ?? and ?? we have used the following additional procedures from a supplementary package AVSD.

- Procedure `phi` provides an isomorphism between a matrix algebra and a Clifford algebra.
- Procedure `radsimplify` simplifies radical expressions in matrices and vectors.
- Procedure `assignL` is needed to write output from a Maple procedure `eigenvects` in a suitable form, it sorts eigenvectors according to the corresponding eigenvalues, and it uses the Gram-Schmidt orthogonalization process, if necessary, to return a complete list of orthogonal eigenvectors.
- Procedure `climinpoly` belongs to the main package CLIFFORD. It computes a minimal polynomial of any element of a Clifford algebra.
- Procedure `makediag` makes a "diagonal" $\Sigma$ matrix consisting of singular values.
- Procedure `embed` embeds the given non-square matrix or a matrix of smaller dimensions into a $2^k \times 2^k$ matrix of smallest $k$ such that it can be mapped into a Clifford algebra.

# APPENDIX B: THIS IS MIKE'S SECOND APPENDIX

In addition to the main package CLIFFORD, Mike has been using his own commands and worksheets.

# VITA

Jane Doe was born December 31, 1975, in Cookeville, TN. While attending junior high in Baxter, she participated in the MATHCOUNTS competition and ranked number 1 at the state level. In 1993 she graduated Magna Cum Laude from Baxter High School with 4.00 GPA ranking 5 out of 156. While still in high school, she completed 32 credit hours from Tennessee Technological University: 13 in calculus, 3 in differential equations, 10 in Spanish and French, and 6 in English. She was a member of Kappa Mu Epsilon, a national honorary society for mathematics students. From 1993 she enrolled as mathematics major at Tennessee Technological University. In 1995 she received S.A. Patil Award for the best mathematics junior. In 1996 she graduated Magna Summa Cum Laude from Tennessee Tech with B.S. in mathematics and minor in computer science. Later that year she was admitted to the M.S. program in mathematics at Tennessee Tech. On the basis of her Master Thesis she has written two papers "On q-Young idempotents" and "On the Hecke algebra embedding into a Clifford algebra" which have been accepted for publication. They will appear in "Advances in Applied Clifford Algebras."