

AN ABSTRACT OF A THESIS

TYPING THESES AND DISSERTATION AT TTU

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Master of Science in Mathematics

Singularity avoidance is of great importance for the proper control of a robotic arm. In a hyper-redundant robot the singularities are much harder to detect and the Clifford algebra can be used to parameterize a singularity space.

TYPING THESES AND DISSERTATION AT TTU

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CERTIFICATE OF APPROVAL OF THESIS

TYPING THESES AND DISSERTATION AT TTU

by

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DEDICATION

This thesis is dedicated to my wife or husband or parents whose encouragement have meant to me so much during the pursuit of my graduate degree and the composition of the thesis. This thesis is dedicated to my wife or husband or parents whose encouragement have meant to me so much during the pursuit of my graduate degree and the composition of the thesis. This thesis is dedicated to my wife or husband or parents whose encouragement have meant to me so much during the pursuit of my graduate degree and the composition of the thesis. This thesis is dedicated to my wife or husband or parents whose encouragement have meant to me so much during the pursuit of my graduate degree and the composition of the thesis. This thesis is dedicated to my wife or husband or parents whose encouragement have meant to me so much during the pursuit of my graduate degree and the composition of the thesis.

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TABLE OF CONTENTS

	Page
LIST OF TABLES	ix
LIST OF FIGURES	x
Chapter	
1. INTRODUCTION	1
1.1 Revisions	1
1.2 Hyper-redundant robots	3
1.2.1 Young Operators Short	4
2. THEORETICAL DEVELOPMENTS	8
2.1 Singularity Problems Shorter	8
2.1.1 Singularity Issues Shorter	8
Special Applications Shorter	8
2.2 Surface Shorter	8
2.2.1 Surface Subsection Shorter	9
2.3 The Main Case	9
2.3.1 Main Case Subsection Shorter	9
3. INCLUDING FIGURES IN THESIS OR DISSERTATION	11
3.1 Developing a Set of Parameters	11
3.2 Plotting the Singularities	11

Appendices	Page
3.3 Interpreting the Plots	12
3.4 Graphics Support	12
3.5 Using the Program to Avoid Singularities	20
REFERENCES	22
APPENDICES	
A: THIS IS MIKE'S FIRST APPENDIX	28
B: THIS IS MIKE'S SECOND APPENDIX	30
VITA	32

LIST OF TABLES

Table	Page
3.1 A sample table	21

LIST OF FIGURES

Figure		Page
3.1	The wave of the future - first time	12
3.2	Tangles representing a braid equation	15
3.3	The wave of the future - second time	17
3.4	File sin.bmp inserted and viewed in PS mode	18
3.5	MATLAB file inserted with colors showing (works only in PCT _E X32 ver. 3.4, and 4.0 GDI mode)	19

CHAPTER 1

INTRODUCTION

1.1 Revisions to `ttuthesis.sty`

In this new `ttuthesis.sty` file dated 12-03-2007 two bugs have been fixed in the definition of `\section` and `\subsection` commands.

- The first bug had to do with the older definitions not allowing for a proper labeling of sections as in `\label{sectionlabel}` and subsections as in `\label{subsectionlabel}`.

That is, when using `\ref{sectionlabel}` and `\label{subsectionlabel}` in the text, only the current chapter number was printed and not the section or the subsection number.

For example, right after the title of the current section, I have inserted `\label{revisions}` so that I could refer to this section later in the text as in `\ref{revisions}` which produces a correct section number 1.1. Likewise, for subsections: The first subsection below has been labeled as `\label{Young}` and it can now be referred as subsection 1.2.1.

Remember that per TTU Graduate School requirements, the subsubsections remain unnumbered: hence, they cannot be referred to with `\ref` command even if they are labeled with `\label` command.

- The second bug had to do with not allowing for a use of an optional parameter with a shorter title as in `\section`, `\subsection`, and `\subsubsection` commands.

If you look at the TeX code of `chapter1.tex` you will find that the title of the current section has been typed in as

```
\section[Revisions]{Revisions to {\tt ttuthesis.sty}}
```

with the `REQUIRED` now parameter showing a shorter title `\section[Revisions]` showing up between the brackets first and the longer title showing up between the braces.

The shorter title appears now in the Table of Contents whereas the longer title appears now in the actual text of the thesis. If you want to have both the same, that's fine, just use for the shorter title the longer title.

This new parameter containing now the short title **MUST** also be used in titles of subsections and subsubsections: If not used, the following error will result when typesetting:

```
./Thesis.lof) [10] (./chapter1.tex
CHAPTER 1.
! Use of \section doesn't match its definition.
1.6 \section{
      Revisions to {\tt ttuthesis.sty}}
?
```

I have caused that error to appear by removing that required now parameter `[Revisions]` from the command `\section` in

```
\section[Revisions]{Revisions to {\tt ttuthesis.sty}}
```

Thus, to summarize, to avoid typesetting errors, you **MUST** now insert these optional parameters with short titles in all commands `\section`, `\subsection`, and `\subsubsection`. If you want the same title to appear in the Table of Contents,

copy the longer title as the shorter title as well. For example, the title of the next section 1.2 is typed up as:

```
\section[Hyper-redundant robots]{Hyper-redundant robots}
\label{robots}
```

In the command `\chapter`, the use of this short title is optional. This is because the command `\chapter` was defined earlier by authors of this style file with greater care and fixing these definitions would require a major rewrite of the file.

If you have any questions, please contact me via email at rablamowicz@tntech.edu.
Cookeville, December 3, 2007

1.2 Hyper-redundant robots

Hyper-redundant robots have many degrees of freedom (DOF) and are sometimes called snake or worm robots. Clifford algebra is a specially described algebra which can resolve rotations and translations without matrix algebra. For a definition of the Clifford algebra see [1] and references therein. Consider the Clifford algebra $Cl_{3,0}$, also denoted as Cl_3 , over \mathbb{R}^3 endowed with a quadratic form $q = \text{diag}(1, 1, 1)$. Let $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ be an orthonormal basis in \mathbb{R}^3 . Then, there is the following famous relation in the Clifford algebra Cl_3 :

$$\mathbf{e}_i \mathbf{e}_j + \mathbf{e}_j \mathbf{e}_i = 2\delta_{i,j} \tag{1.1}$$

where $\delta_{ij}, i, j = 1, 2, 3$ is the Kronecker delta function. Notice that relation (1.1) reduces to

$$\mathbf{e}_i^2 = 1, \quad i = 1, 2, 3. \quad (1.2)$$

The aim of this work is to provide a mechanism for breaking up the ordinary spinor representation of $\mathcal{C}\ell_{n,n}$ into tensor products of smaller representations using appropriate Young operators constructed as Clifford idempotents. This means that a suitable Clifford algebra is used as a carrier space for various tensor product representations.

1.2.1 Young Operators Long

The Young operators for various Young diagrams provide a set of idempotents which decompose the unity $\mathbf{1}$ of $\mathcal{C}\ell_{n,n}$ as

$$\mathbf{1} = Y^{(\lambda_1)} + \dots + Y^{(\lambda_n)}, \quad (1.3)$$

where (λ_i) is a partition of n characterizing the appropriate Young tableau, that is, a Young diagram (frame) with an allowed numbering. Let $Y_{i_1, \dots, i_n}^{(\lambda_i)}$ denote a Young tableaux where (λ_i) is an ordered partition of n and i_1, \dots, i_n is an allowed numbering of the boxes in the Young diagram corresponding to (λ_i) as in [23, 30]. Furthermore, these Young operators are mutually annihilating idempotents

$$Y^{(\lambda_i)} Y^{(\lambda_j)} = \delta_{\lambda_i \lambda_j} Y^{(\lambda_j)}. \quad (1.4)$$

It appears natural to ask if these Young operators can be used to give representations of the symmetric group *within* the Clifford algebraic framework. The representation

spaces which appear as a natural outcome of the embedding of the symmetric group, and its representations can then be looked at as multi-particle spinor states. However, these might not be spinors of the full Clifford algebra.

In order to be as general as possible, in the following, not only the representations of the symmetric group will be considered, but also of the Hecke algebra $H_{\mathbb{F}}(n, q)$. The Hecke algebra is the generalization of the group algebra of the symmetric group by adding the requirement that transpositions t_i of *adjacent* elements $i, i + 1$ are no longer involutions s_i . Equation $t_i^2 = (1 - q)t_i + q$ reduces to $s_i^2 = \mathbf{1}$ in the limit $q \rightarrow 1$.

Hecke algebras are ‘truncated’ braids since a further relation (see (1.5) below) is added to the braid group relations as in [4]. A detailed treatment of this topic with important links to physics may be found, for example, in [21, 38] and in the references of [12].

The defining relations of the Hecke algebra will be given according to Bourbaki [9]. Let $\langle \mathbf{1}, t_1, \dots, t_n \rangle$ be a set of generators which fulfill these relations:

$$t_i^2 = (1 - q)t_i + q, \quad (1.5)$$

$$t_i t_j = t_j t_i, \quad |i - j| \geq 2, \quad (1.6)$$

$$t_i t_{i+1} t_i = t_{i+1} t_i t_{i+1}. \quad (1.7)$$

Then their algebraic span is the Hecke algebra. Since the results in this thesis will be compared with those of King and Wybourne [29] – hereafter denoted by KW – one needs to provide a transformation to their generators g_i , namely $g_i = -t_i$, which results in a new quadratic relation

$$g_i^2 = (q - 1)g_i + q \quad (1.8)$$

while the other two remain unchanged. However, this small change in sign is responsible for great differences especially in the q -polynomials occurring in both formulas. One immediate consequence is that this transformation interchanges symmetrizers and antisymmetrizers. In particular, this replacement connects Formula (3.4) in KW with full symmetrizers while Formula (3.3) in KW gives full antisymmetrizers. Finally, the algebra morphism ρ which maps the Hecke algebra into the even part of an appropriate Clifford algebra can be found in [12].

Let $\{\mathbf{1}, \mathbf{e}_1, \dots, \mathbf{e}_{2n}\}$ be a set of generators of the Clifford algebra $\mathcal{Cl}(B, V)$ where the vector space $V = \{\mathbf{e}_i\} = \langle \mathbf{e}_i \rangle$ is endowed with a non-symmetric $2n \times 2n$ bilinear form $B = [B(\mathbf{e}_i, \mathbf{e}_j)] = [B_{i,j}]$ defined as

$$B_{i,j} := \begin{cases} 0, & \text{if } 1 \leq i, j \leq n \text{ or } n < i, j \leq 2n, \\ q, & \text{if } i = j - n \text{ or } i - 1 - n = j, \\ -(1 + q), & \text{if } i + 1 = j - n \text{ or } i = j + 1 - n, \\ -1, & \text{if } |i - j - n| \geq 2 \text{ and } i > n, \\ 1, & \text{otherwise.} \end{cases} \quad (1.9)$$

The most general case would have $\nu_{ij} \neq 0$ in the last line of (1.9). For example, when $n = 4$, then

$$B = \begin{pmatrix} 0 & 0 & 0 & 0 & q & -1-q & 1 & 1 \\ 0 & 0 & 0 & 0 & -1-q & q & -1-q & 1 \\ 0 & 0 & 0 & 0 & 1 & -1-q & q & -1-q \\ 0 & 0 & 0 & 0 & 1 & 1 & -1-q & q \\ 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 \\ q & 1 & 1 & -1 & 0 & 0 & 0 & 0 \\ -1 & q & 1 & 1 & 0 & 0 & 0 & 0 \\ -1 & -1 & q & 1 & 0 & 0 & 0 & 0 \end{pmatrix}. \quad (1.10)$$

The bilinear form B in (1.10) is our particular choice that guarantees that the following equations hold:

$$\rho(t_i) = b_i := \mathbf{e}_i \wedge \mathbf{e}_{i+n}, \quad (1.11)$$

$$b_i b_j = b_j b_i, \quad \text{whenever } |i - j| \geq 2, \quad (1.12)$$

$$b_i b_{i+1} b_i = b_{i+1} b_i b_{i+1}. \quad (1.13)$$

This shows ρ to be a homomorphism of algebras implementing the Hecke algebra structure in the Clifford algebra $\mathcal{Cl}(B, V)$. One knows from [12] that ρ is not injective, and that its kernel contains all Young diagrams which are not L-shaped (that is, diagrams with at most one row and/or one column). The first instance, however, when this kernel is non-trivial, occurs in S_4 where the partition $4 = (2, 2)$ gives a Young diagram of square form which is not L-shaped.

Testing section 2.3 here.

CHAPTER 2

THEORETICAL DEVELOPMENTS

This chapter explains the design technique developed in this research project. The first section bla bla bla. Ultimately the equation

$$E = m * c^2 \tag{2.1}$$

$$E = m * 9 * 10^{16} \tag{2.2}$$

In Section 2.1 we will show something. In Section 2.2 we will show something else, while in Section 2.3 we will discuss a new case.

In Subsection 2.1.1 we will show something. In Subsection 2.2.1 we will show something else, while in Subsection 2.3.1 we will discuss a new case.

2.1 Singularity Problems Longer

At a singularity, an infinite joint velocity is required to move the end effector in a particular direction as in Equation 2.2 [32]. Also near the singularity large forces can be found that can damage the robot or an object being worked on. Bla bla bla.

2.1.1 Singularity Issues Longer

Special Applications Longer.

2.2 Surface Parameterization Longer

Clifford algebras of the $C\ell^+$ can be used to bla bla bla.

2.2.1 Surface Subsection

2.3 The case of $H_{\mathbb{F}}(2, q)$ and S_2

Beginning with $H_{\mathbb{F}}(2, q)$ which reduces to S_2 in the limit $q \rightarrow 1$. $H_{\mathbb{F}}(2, q)$ is generated by $\{\mathbf{1}, b_1\}$. Thus, having only one q -transposition, from which a q -symmetrizer $R(12)$ and a q -antisymmetrizer $C(12)$ can be calculated.

2.3.1 Main Case

Notice that in the limit $q \rightarrow 1$ the following relations for a set of new generators defined as $s_i := \lim b_i$ when $q \rightarrow 1$:

- (i) $s_i^2 = 1$,
- (ii) $s_i s_j = s_j s_i$, whenever $|i - j| \geq 2$,
- (iii) $s_i s_{i+1} s_i = s_{i+1} s_i s_{i+1}$,
- (iii)' $(s_i s_{i+1})^3 = \mathbf{1}$.

Property (iii)' follows from the fact that $s_i^2 = 1$ and $s_i^{-1} = s_i$. This is a presentation of the symmetric group according to Coxeter-Moser [7]. Now it is an easy matter to show that (ii) is valid for transpositions, and that (iii) can be calculated graphically using tangles as in Figure 1.

Definition 1. *An associative algebra over a field \mathbb{F} with unity 1 is the Clifford algebra $Cl(Q)$ of a non-degenerate quadratic form Q on a vector space V over \mathbb{F} if it contains V and $\mathbb{F} = \mathbb{F} \cdot 1$ as distinct subspaces so that*

1. $\mathbf{x}^2 = Q(\mathbf{x})$ for any $\mathbf{x} \in V$,
2. V generates $Cl(Q)$ as an algebra over \mathbb{F} ,
3. $Cl(Q)$ is not generated by any proper subspace of V .

Theorem 1. *Suppose that $1 \leq p < (1 - \kappa)^{-1}$. If $u \in L^p(\mathbb{R}^n, \mathbb{R})$ and $u \geq 0$, then*

$$|\mathcal{I}u(x)| \leq A_*(k, p)[\mathcal{M}_*u(x)]^{1-(1-\kappa)p} \|u\|_p^{(1-\kappa)p}, \quad x \in \mathbb{R}^n. \quad (2.3)$$

Proof. Since $u \geq 0$, it is clear that

$$|k * u(x)| \leq \max\{|k_+ * u(x)|, |k_- * u(x)|\}. \quad (2.4)$$

By applying Theorem 1 to k_+ and k_- one gets

$$|k_{\pm} * u(x)| \leq A(k_{\pm}, p)[\mathcal{M}_{\pm}u(x)]^{1-(1-\kappa)p} \|u\|_p. \quad (2.5)$$

It remains to combine (2.4) and (2.5) with (??) and (??).¹

The reader can easily check that (2.3) is sharp. Moreover, if it is assumed that the kernel k is an odd function, then $X_- = (-1)X_+$ and

$$\text{vol}(X_+) = \text{vol}(X_-) = \frac{1}{2}\text{vol}(X);$$

hence,

$$A_*(k, p) = 2^{-\kappa} A(k, p). \quad (2.6)$$

□

¹These last two calls to references have resulted in (??) and (??) showing, and in an error message in your paper's log file that is automatically generated by PCT_{EX}32. This is because references have been made to non-existing labels.

CHAPTER 3

INCLUDING FIGURES IN THESIS OR DISSERTATION

The math required to find the singularity space can be quite lengthy and errors are easily made. Therefore a computer program called CLIFFORD (see [3]) is used. In addition, it is useful to visualize the singularity space via a graph which can be generated by Maple. Thus, it is also useful to know how to include plots and figures in a \LaTeX document.

The following will show how to include plots in your \LaTeX document. For more information on basic \LaTeX commands see [24] and for more advanced features of $\mathcal{A}\mathcal{M}\mathcal{S}\text{-}\text{\LaTeX}$ see [22].

3.1 Developing a Set of Parameters

CLIFFORD can perform computations in any Clifford algebra; in particular, it can compute with quaternions. Quaternions give a convenient representation of rigid motions in a three dimensional real space \mathbb{R}^3 .

3.2 Plotting the Singularities

One can control vertical spacing before and after a figure with `\vskip` commands. For example, the following code produces Figure (3.1).

```
\begin{figure}[tb]
\vskip3pt %controls vertical space before the figure
\centereps{3in}{3in}{plot03.eps}
\caption{The wave of the future - first time}
\label{fig:sin}
\vskip0.5pt %controls vertical space after the figure
\end{figure}
```

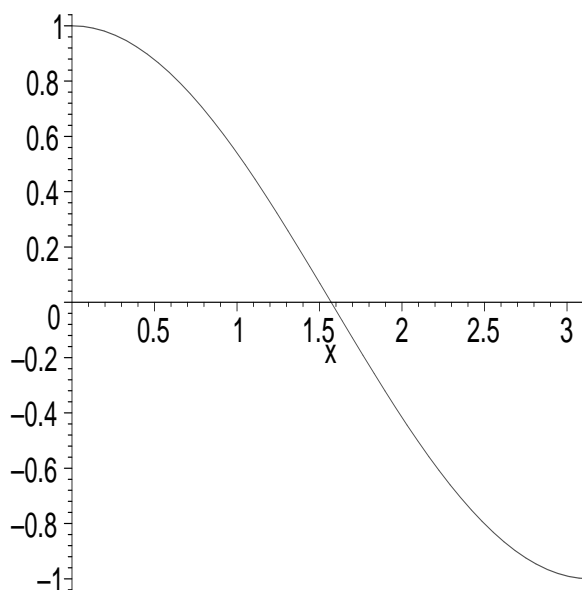


Figure 3.1. The wave of the future - first time

As it can be seen from the code that produces Figure 3.1, this is simple.

3.3 Interpreting the Plots

Interpreting the plots is almost impossible (joke!). However, see the following examples how plots in various formats can be inserted. Help page from PCT_EX32 gives the following information.

3.4 Graphics Support

Graphic images may be included in the documents created with PCT_EX32 by using the `\special` command. However, a simple usage below creates a problem. That's why it is better to use other L^AT_EX commands which give better control over the placement of the pictures.

PCT_EX32 supports four categories of graphic formats: the Windows bitmap, or BMP, format; the Encapsulated Postscript, or EPS, graphic format; the Windows

Metafile(including Enhanced), or WMF, format; and the Postscript, or PS, graphic format. The simplest usage may, however, cause problems (see below).

```
\special{bmp:<path+filename> x=n y=n}
\special{eps:<path+filename> x=n y=n}
\special{wmf:<path+filename> x=n y=n}
\special{ps:<path+filename> x=n y=n}
```

The following example may look very bad depending on where it is put in the text. Notice that the Graduate School requires that any table or figure be placed at the top or at the bottom of a page. Thus, one must use

```
\begin{figure}[tb]
...
\end{figure}
```

environment with the parameters `[tb]` and the actual `\special` command placed inside. This way the figure that is to be inserted with the command `\special` will become a “float”, that is, an object whose position in the text will be determined by the compiler. This position will either be the top of the page, which is specified by the letter `t` or the bottom of the page, which is specified by the letter `b` in `[tb]`. Notice also that extra spacing is added by the line `\vskip3in` inside the figure environment for proper spacing.

```
\begin{figure}[tb]
\vskip3in
\special{bmp:sin.bmp x=3in y=3in}
\end{figure}
```

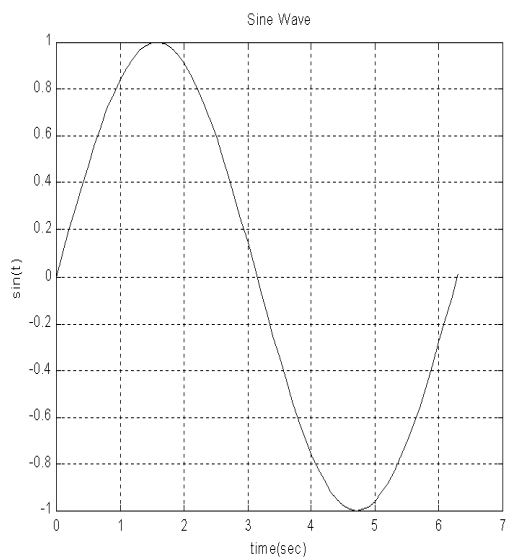
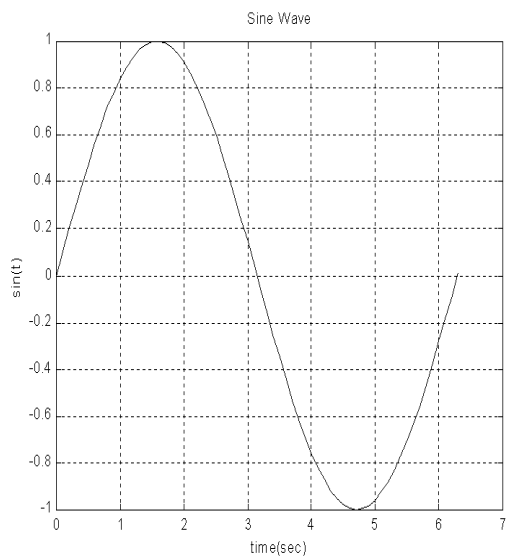
A sample `\special` command for a BMP graphic image might look like:

```
\special{bmp:c:/figures/picture.bmp x=5in y=3in}
```

For Windows Enhanced Metafiles, simply use the WMF `\special` command such as:

```
\special{wmf:c:/figures/myempic.emf x=5in y=3in}
```

When specifying paths to the files, use `"/` instead of `"\` to specify the subdirectories.



The BMP, EPS, WMF, or PS graphic image will be scaled to the value of x and y . The dimensions "cm" and "in" are accepted. One needs to leave enough vertical space before the `\special` command for the image. So, try one more time to plot with the command `\special`. The code used to print next figure

```
\begin{figure}[tb]
\vskip3in
\special{bmp:sin.bmp x=3in y=3in}
```

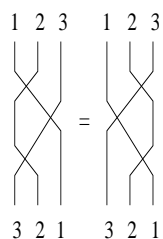


Figure 3.2. Tangles representing a braid equation

```
\end{figure}
\noindent
```

where `\noindent` is needed so that the text following the figure, i.e., “To print the above figure...”, is not indented. Included in the above code is a vertical skip before the picture equal to 3 inches which is the height of the picture specified by `y=3in`. Notice, that the picture has not been centered and that it has no caption. The latter is the reason why it will not appear in the list of figures earlier in this document. Only figures with a caption (see next) will appear in the list of figures.

However, try the following which gives much better control where the picture will appear. Notice that command `\psfig` from the `psfig` package is used. Notice also that the placement of the figure is controlled by the option `[htb]`. Letter `h` in `[htb]` means “place this figure here, if possible,” letter `t` means “place this figure at the top of the page, if possible”, while letter `b` means “place this figure at the bottom of the page, if possible.” There is one more option `p` which means “place this figure at the end of the document on a page of floats (not a text page).”

```
\begin{figure}[tb]
\centerline{\psfig{figure=braid.eps,height=3.0cm,width=2.0cm}}
\caption{Tangles representing a braid equation}
\label{fig:tangles}
\end{figure}
```

The above code gives Figure 3.2) below. Notice also that this figure has appeared in the list of all figures at the beginning of the document. Most importantly, notice that

the spacing before and after the figure is done automatically and `\vskip` or `\noindent` commands for proper spacing and justification do not need to be used. Finally, there is a caption, and the figure is automatically numbered. Its caption appears also in the list of figures. Also included is `\label` command in the line `\label{fig:tangles}` and from now on this figure can be referred to as as Figure 3.2 by using `\ref` command this way: to refer to Figure 3.2 just type `Figure~\ref{fig:tangles}`. The `~` symbol is not printed but it prevents the number 3.2 from being separated from the word “Figure” at the end of the line (notice that the word “number” above is separated from 3.2).

The macros `SETBMP`, `SETEPS`, `SETWMF`, and `SETPS` are provided to assist in including graphic images. To use the `SETBMP`, `SETEPS`, `SETWMF`, or `SETPS` macros, use the `\input` command to include the macro.¹ These macros define the `\centerbmp`, `\centereps`, `\centerwmf`, and `\centerps` commands; and the `\setbmp`, `\seteps`, `\setwmf`, and `\setps` commands respectively.

```
\setbmp{i}{x}{y}{<path+filename>}
\seteps{i}{x}{y}{<path+filename>}
\setwmf{i}{x}{y}{<path+filename>}
\setps{i}{x}{y}{<path+filename>}
```

In the above examples, `i` designates the indentation of the graphic image. `x` designates the width of the image. `y` designates the height of the image.

```
\centerbmp{x}{y}{<path+filename>}
\centereps{x}{y}{<path+filename>}
\centerwmf{x}{y}{<path+filename>}
\centerps{x}{y}{<path+filename>}
```

The `\centerbmp`, `\centereps`, `\centerwmf` or `\centerps` command centers the image horizontally on the page. `x` is the width and `y` is the height of the graphic image. The units “cm” and “in” are accepted. Indicate the desired amount of blank space

¹Notice that these input commands have been included already in `Thesis.tex` file.

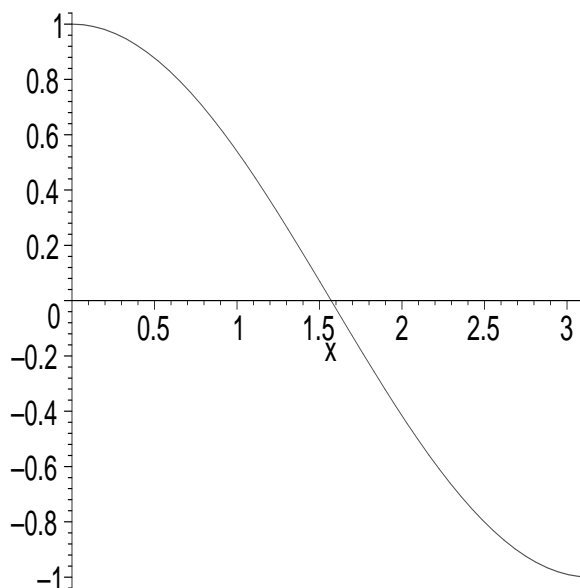


Figure 3.3. The wave of the future - second time

before and after the command with `\vskip`. One can also add labels with `\label` command and captions with `\caption` command.

For example, the following code

```
\begin{figure}[tb]
\centereps{3in}{3in}{plot03.eps}
\caption{The wave of the future - second time}
\label{fig:eps1}
\end{figure}
```

produces Figure (3.3). Likewise, the following code inserts file of type BMP:

```
\begin{figure}[tb]
\centerbmp{3in}{3in}{sin.bmp}
\caption{File sin.bmp inserted and viewed in PS mode}
\label{fig:bmp1}
\end{figure}
```

which should give a figure here but it is not right here... Where is it? Its location in the text is somewhat fluid and it is determined by the \LaTeX code. Objects whose location depends on the surrounding text are called “floats.” The figure could be at the bottom of the page or it could be on the next page! One can also insert

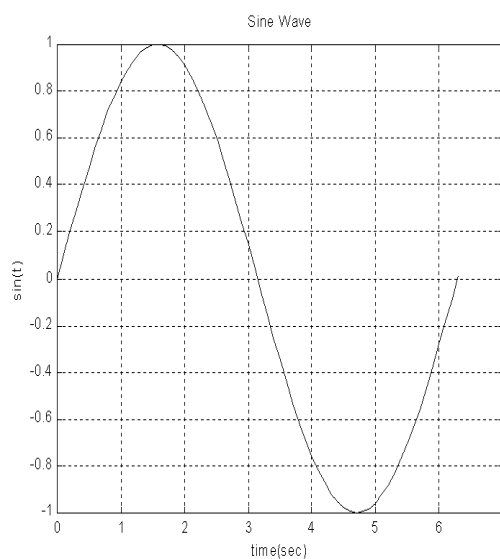


Figure 3.4. File sin.bmp inserted and viewed in PS mode

pictures/figures created with MATLAB that have extension .emf as follows. The code

```
\begin{figure}[tb]
\centerwfmf{3in}{3in}{sin.emf}
\caption{MATLAB file inserted with colors showing (works only in
PC{\TeX}32 ver. 3.4, and 4.0 GDI mode)}
\label{fig:emf1}
\end{figure}
```

works in PCT_EX32 ver. 3.4 and in PCT_EX32 ver. 4.0 in GDI mode but not in PS mode!. The code above produces Figure 3.5. Notice that in the labels used in the pictures above, e.g., `\label{fig:emf1}`, a counter called `fig` has been used. This way one can refer in the text to each of the above figures with `\ref` command as follows. To refer to the BMP picture just type in (`\ref{fig:bmp1}`). For example: “In picture (3.4) a sine wave can be seen”.

Remark: MATLAB saves pictures in files with extension .emf, and not .wmf: there is no .wmf format! That’s why the command

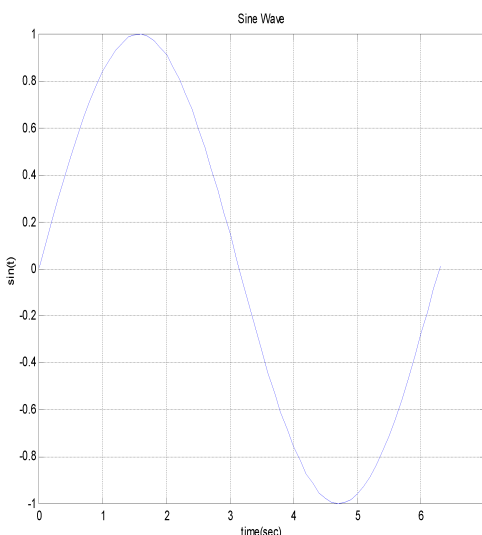


Figure 3.5. MATLAB file inserted with colors showing (works only in PCT_EX32 ver. 3.4, and 4.0 GDI mode)

```
\centerwmf{3in}{3in}{sin.emf}
```

works in PCT_EX32 version 3.4. However, version 4.0 of PCT_EX32 can be set in either Postscript mode PS or Windows mode GDI.² PCT_EX32 will display an error when trying to preview the dvi file saying

```
dvips: Invalid image file sin.emf
```

after `\centerwmf{3in}{3in}{sin.emf}` has been used. In order to avoid this error, change PS mode to GDI (see the footnote at the bottom of this page).

However, once changed from PS mode to GDI mode in PCT_EX32, Postscript images in the .dvi file will not be seen once the document is composed. A suggestion is to use one type of picture files, if possible. One should also know that even though

²To find out in which mode PCT_EX32 runs on your machine, look at the bottom menu bar to the right of the “Stop” button: it will say PS or GDI accordingly. If you use .emf files in your document, you will not be able to correctly compose your document in ver. 4.0 of PCT_EX32 if it runs in the Postscript mode indicated by PS. You will need to change from PS to GDI mode as follows: go to the top menu bar in PCT_EX32 and click on: Settings, Default Settings, Advanced and change from Postscript to Windows/GDI. You will need to restart PCT_EX32.

figures are not displayed in the dvi viewer, they still may print correctly. This is true of PCT_EX32 ver. 3.4 which is unable to display any figures in its dvi viewer but it will show at least the caption and the bounding box for PS figures. Version 4.0 of PCT_EX32 will not be able to display pictures in certain formats depending on in which mode it is run: PS or DVI. Yet, it will most likely print correctly.

3.5 Using the Program to Avoid Singularities

It is so simple to use this program. As seen in Table 3.1. To display this table, the following code will be used.

```
\begin{table}[tb]
  \caption{A sample table} \label{tab}
  \vspace{0.2in}
  \arrayrulewidth 1pt
  \doublerulesep 5pt
  \begin{center}
    \begin{tabular}{ccc} \hline \hline
      Header 1 & Header 2 & Header 3 \\ \hline
      1.000 & 2.000 & 3.000 \\ \hline
      1.275 & 2.500 & 3.250 \\ \hline
      1.500 & 2.875 & 3.450 \\ \hline
    \end{tabular}
  \end{center}
\end{table}
```

Notice, that again one must make sure that the table will be placed at the top or at the bottom of a page. This is accomplished by using parameters [tb] as in `\begin{table}[tb]`. Then, the table will be placed centrally on a page, either at the top or at the bottom.

Table 3.1. A sample table

Header 1	Header 2	Header 3
1.000	2.000	3.000
1.275	2.500	3.250
1.500	2.875	3.450

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APPENDICES

APPENDIX A: THIS IS MIKE'S FIRST APPENDIX

In addition to the main package CLIFFORD, in Sections ?? and ?? we have used the following additional procedures from a supplementary package AVSD.

- Procedure `phi` provides an isomorphism between a matrix algebra and a Clifford algebra.
- Procedure `radsimplify` simplifies radical expressions in matrices and vectors.
- Procedure `assignL` is needed to write output from a Maple procedure `eigenvects` in a suitable form, it sorts eigenvectors according to the corresponding eigenvalues, and it uses the Gram-Schmidt orthogonalization process, if necessary, to return a complete list of orthogonal eigenvectors.
- Procedure `climpinpoly` belongs to the main package CLIFFORD. It computes a minimal polynomial of any element of a Clifford algebra.
- Procedure `makediag` makes a “diagonal” Σ matrix consisting of singular values.
- Procedure `embed` embeds the given non-square matrix or a matrix of smaller dimensions into a $2^k \times 2^k$ matrix of smallest k such that it can be mapped into a Clifford algebra.

APPENDIX B: THIS IS MIKE'S SECOND APPENDIX

In addition to the main package CLIFFORD, Mike has been using his own commands and worksheets.

VITA

Jane Doe was born December 31, 1975, in Cookeville, TN. While attending junior high in Baxter, she participated in the MATHCOUNTS competition and ranked number 1 at the state level. In 1993 she graduated Magna Cum Laude from Baxter High School with 4.00 GPA ranking 5 out of 156. While still in high school, she completed 32 credit hours from Tennessee Technological University: 13 in calculus, 3 in differential equations, 10 in Spanish and French, and 6 in English. She was a member of Kappa Mu Epsilon, a national honorary society for mathematics students. From 1993 she enrolled as mathematics major at Tennessee Technological University. In 1995 she received S.A. Patil Award for the best mathematics junior. In 1996 she graduated Magna Summa Cum Laude from Tennessee Tech with B.S. in mathematics and minor in computer science. Later that year she was admitted to the M.S. program in mathematics at Tennessee Tech. On the basis of her Master Thesis she has written two papers "On q -Young idempotents" and "On the Hecke algebra embedding into a Clifford algebra" which have been accepted for publication. They will appear in "Advances in Applied Clifford Algebras."